

# The Fourier Transform And Milankovitch's Theory of Glacial Cycles

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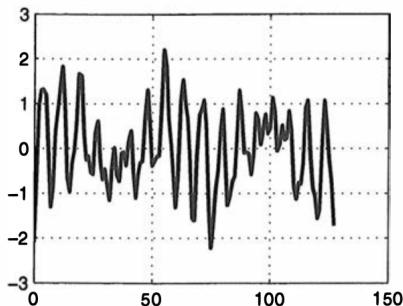
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# Outline

- 1 The Fourier Transform
- 2 Power Spectrum
- 3 Milankovitch's theory of glacial cycles
- 4 Conclusion

## What is Fourier Analysis?

- A tool for extracting information from a time series or signal.
- It enables us to find the periods and relative strengths of the individual components from a complex signal.
- Trigonometric interpolation: write the signal as a linear combination of sines and cosines.



## Defining the Problem

- Suppose we are given a sequence  $x_j$ ,  $j = 0, 1, \dots, N - 1$  of  $N$  data points that have been obtained by sampling an unknown function  $x : t \rightarrow x(t)$  at times  $t = 0, 1, \dots, N - 1$ .

- Assuming  $x$  can be written as a sum of sines and cosines, we seek coefficients  $c_k$  such that we can write

$$x(t) = \sum_{n=0}^{N-1} c_k \cdot e^{ikt2\pi/N}$$

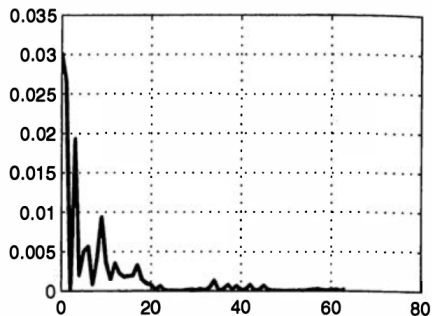
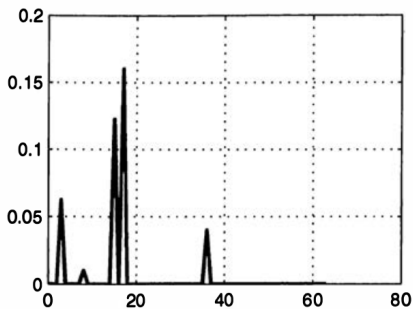
## Discrete Fourier Transform

- Since we want the transform for a finite interval, we can evaluate this expression at  $t = 0, 1, \dots, N - 1$ . This gives us:  
$$x_j = \sum_{n=0}^{N-1} c_n \cdot e^{ikj2\pi/N}$$
- This system is summarized by considering the  $x_j$ 's,  $x_0, x_1, \dots, x_{N-1}$  and the  $c_k$ 's,  $c_0, c_1, \dots, c_{N-1}$  as entries to the vectors  $x$  and  $c$  respectively. We obtain:  $x = F_N c$
- Here  $F_N$  is the matrix where  $e^{ikj2\pi/N}$  is evaluated from  $j = 0$  to  $j = N - 1$ .
- The Fourier matrix is invertible. There is a shortcut to compute the DFT called the Fast Fourier Transform.

## What is the Power Spectrum?

- The power spectrum is a graph that plots amplitude against frequency. The amplitude is obtained using  $(|c_k|^2)$  and plotted against  $2k\pi/T$ .
- It is particularly useful in analyzing the data obtained from the Fourier transform. Particularly, a spike on a power spectrum graph represents that the corresponding frequency has a prominent periodic component in the signal.
- It is useful to notice trends or anomalies in data collected over a long interval, such as a millennia.

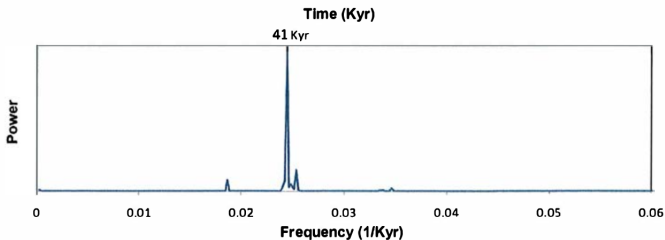
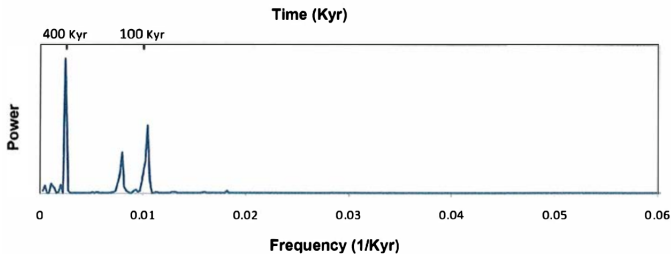
## What is the Power Spectrum? Continued



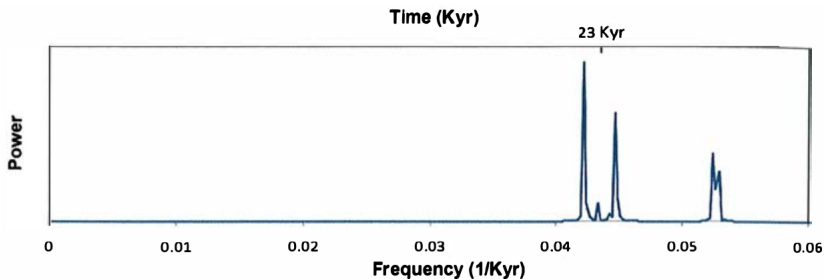
# The Theory of Glacial Cycles

- Past glacial cycles are correlated to incoming solar energy.
- Glacial cycles depend on three parameters:
  - Eccentricity (shape of the elliptical orbit of the Earth); 400 Kyr and 100 Kyr
  - Obliquity (tilt of the earth); 41 Kyr period
  - Precession (changes in axis of rotation; more wobbly vs. smooth); 23 Kyr period
- It is useful to notice trends or anomalies in data collected over a long interval, such as a millennia.

# The Theory of Glacial Cycles Continued



## The Theory of Glacial Cycles Continued



## Conclusion

- The DFT is a way to "assign" a linear combination of sines and cosines to data that has no clear pattern.
- The power spectrum is a graph that uses the amplitudes found by the DFT and plots it with the corresponding frequencies.
- Milankovitch's Theory of Glacial Cycles assigns three orbital parameters as heralds of changing glaciation. The periods of these parameters are found using the power spectrum.

## References

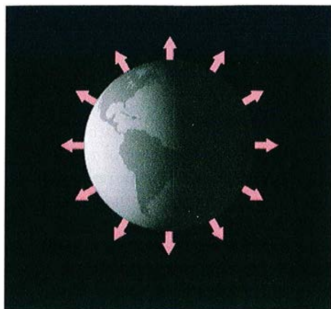
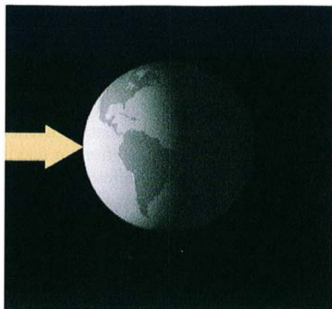
- Kaper, Hans, and Hans Engler. Mathematics and Climate. Edited by Hans Kaper, Society for Industrial and Applied Mathematics, 2013.
- Images used with permission from author.

## Fast Fourier Transform

- The Fourier Transform is difficult to compute and has an  $O(n^2)$  time complexity. A different way to compute the DFT is the FFT which has a time complexity of  $O(n \log(n))$ .
- The FFT does this by splitting the components of vector  $x$  into even and odd indices and then proceeding from there.

## Climate models

- System-level approach: whole vs. parts.
- Energy balance model



# The Earth's Climate

- Climate is "averaged weather".
- Earth's climate is made up of different components:
  - the atmosphere
  - the hydrosphere (bodies of water)
  - the cryosphere (snow and ice)
  - the lithosphere (land surface)
  - the biosphere (living things)