1. (10 points) Use complex analysis to compute
\[ I = \int_0^\infty \frac{\cos ax - \cos bx}{x^2} \, dx, \]
where \(a\) and \(b\) are positive constants. Hint: Start by considering \(\frac{\cos ax - \cos bx}{x^2}\) as the real part of an appropriate complex valued function.

2. (10 points) Let \(f(z) = \sum_{n=0}^{\infty} c_n z^n\) be analytic and one-to-one in \(|z| < 1\) with real part \(u(z)\) and imaginary part \(v(z)\). For \(0 < r < 1\), let \(D_r\) be the disc \(|z| < r\). Prove that the area \(A_r\) of \(f(D_r)\) is finite and is given by the following formula:
\[ \int \int_{f(D_r)} \, dudv = \pi \sum_{n=1}^{\infty} n|c_n|^2 r^{2n}. \]

3. (10 points) Let \(a_n(z)\) be a sequence of analytic functions on an open set \(\Omega\) such that \(\sum_{n=0}^{\infty} |a_n(z)|\) converges uniformly on its compact subsets. Show that \(\sum_{n=0}^{\infty} |a'_n(z)|\) also converges uniformly on compact subsets of \(\Omega\).

4. (10 points) Show that if \(f(z)\) and \(g(z)\) are holomorphic functions on an open and connected set \(\Omega\) such that \(f(z)g(z) = 0\), then either \(f(z)\) or \(g(z)\) is identically zero.

5. (10 points) Give the Laurent expansion of \(f(z) = \frac{1}{z(z-1)}\) in each of the following two annuli
   (i) \(\{z : 0 < |z| < 1\}\), (ii) \(\{z : 1 < |z|\}\)

6. (10 points) Find a conformal map from the intersection \(D\) of \(|z - i| < 2\) and \(|z + i| < 2\) to the upper half plane.

7. (10 points) Show that \(z = 0\) is an essential singularity for the function \(f(z) = e^{\frac{1}{z^m z}}\)