**1.** Suppose $(X, d)$ is a metric space. State criteria for continuity of a function $f : X \to X$ in terms of

**i)** open sets,
**ii)** $\epsilon$'s and $\delta$'s, and
**iii)** convergent sequences.

Then prove that (iii) implies (i).

**2.** Prove that every compact, Hausdorff topological space is normal.

**3.** Write $Y$ for the interval $[0, \infty)$, equipped with the usual topology. Find, with proof, all subspaces $Z$ of $Y$ which are retractions of $Y$.

**4.** State, without proof, a criterion in terms of the fundamental group for a covering map $p : \tilde{X} \to X$ to be regular. Give an example of a covering map that is not regular, and show explicitly how it fails to satisfy this criterion.

**5.** Start with the unit disk $\mathbb{D}^2$ and identify points on the boundary if their angles, thought of in polar coordinates, differ by a multiple of $\frac{\pi}{2}$. Let $X$ be the resulting space. Use van Kampen's theorem to compute $\pi_1(X, *)$.

**6.** Let $S^2 \to \mathbb{RP}^2$ be the universal covering map. Is this map null-homotopic? Give a proof of your answer.

**7.** Compute the homology of the space $X$ obtained by attaching a Mobius band to $\mathbb{RP}^2$ via a homeomorphism of its boundary circle to the standard $\mathbb{RP}^1$ in $\mathbb{RP}^2$.

**8.** Let $X = S^2/\{p_1 = \cdots = p_k\}$ be the topological space obtained from the 2-sphere by identifying $k$ distinct points on it ($k \geq 2$).

**a)** Find the fundamental group of X.
**b)** Find the Euler characteristic of X.
**c)** Find the homology groups of X.

**9.** Let $M$ and $N$ be finite CW-complexes.

**a)** Describe the cellular structure of $M \times N$ in terms of the cellular structure of $M$ and $N$.
**b)** Show that the Euler characteristic of $M \times N$ is the product of the Euler characteristics of $M$ and $N$.