Numerical Analysis Preliminary Examination
Spring, 2011

NAME____________________ SCORE ____________________

Instruction: Do all problems and show all your work.

[1] (10pts) Consider the Steffensen method for nonlinear equation $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)}.$$

Show that this is quadratically convergent under suitable hypotheses. Please state the hypotheses and give your proof.

[2] (10pts) Let x_k and x_{k+1} be two successive iterates when Newton's method is applied to find the zeros of a polynomial p of degree n. Show that there is a zero of p within distance $n|x_k - x_{k+1}|$ of x_k.

[3] (10pts) Suppose that a matrix A is diagonally dominent. Show that the Gauss-Jacobi's method for $Ax = b$ converges.

[4] (10pts) Suppose that A is weakly diagonally dominant and is irreducible. Show that the Gauss-Jacobi's method for $Ax = b$ also converges.

[5] (10pts) Find a Householder's transformation to convert the following vector $\mathbf{v}$ into $[0, 0, 0, \alpha]^T$ with α being the norm of the vector $\mathbf{v}$:

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

[6] (10pts) Explain how one can find an orthonormal matrix Q and a lower triangular matrix L for a given square matrix A such that A can be factored into $A = QL$.

[7] (10pts) Define the QR iterative method for numerical solution of eigenvalues of symmetric matrix A. Explain each step.

[8] (10pts) Define the least square data fitting problem and explain how to use the SVD method to solve the least square data fitting problem.

[9] (10pts) Let $a = x_0 < x_1 < \cdots < x_n < x_{n+1} = b$ be a partition of $[a, b]$. For $f \in C[a, b]$, let S_f be the C^2 natural cubic interpolatory spline of f, i.e.,

$$S_f(x_i) = f(x_i), i = 0, 1, \cdots, n+1, S_f''(a) = 0 = S_f''(b),$$

Suppose that $f \in C^2[a, b]$. Show that

$$\int_a^b |S_f''(x)|^2 dx \le \int_a^b |f''(x)|^2 dx.$$

[10] (10pts) Let $B_i^n(x)$ be B-spline of order n over knots $x_i, x_{i+1}, \cdots, x_{i+n}$. Show that $B_i(x) \ge 0$ and

$$\sum_i B_i^n(x) = 1.$$