UGA High School Varsity Math Tournament
October 26, 2024

Ciphering Round

Time: 2 minutes per problem
Length: 10 problems

Max Score 100 points
10 points for a correct answer.

Do NOT open this booklet.

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Problem 1.

Last Name

ID

Answer

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Problem 1. Solve for a:

$$a^3 + 2^a = 2024.$$

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Problem 2.

Last Name

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Problem 2. What is the number of prime numbers between 100 and 1000 whose digits are in arithmetic progression? Recall that numbers are said to form an arithmetic progression if the difference between consecutive numbers is constant.

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Problem 3.

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Answer

Problem 3. How many integer points lie on the closed line segment joining $(0, 0)$ to $(20, 24)$?
The point (x, y) is an integer point if both x and y are integers, e.g. $(0, 0)$. A line segment is said to be closed when it includes both endpoints.

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Problem 4.

Last Name

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Answer

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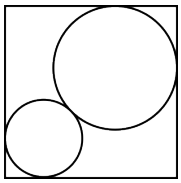
Last Name

ID

Answer

Problem 4. Two circles are tangent internally at two points of a square of side one. The diameter of the larger circle is 11 times that of the smaller one. Given that the circles are mutually tangent, what is the sum of their radii? Write your answer in the form $a + b\sqrt{c}$ where a, b and c are integers.



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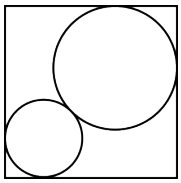

Problem 5.

Last Name

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Answer

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Answer

Problem 5. How many digits does the number $4^{16}5^{25}$ have (in base 10)?

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Problem 6.

Last Name

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Answer

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Answer

Problem 6. A multiset is a set where elements can be repeated, e.g. $\{2, 2, 5, 13\}$ is a multiset with 4 elements. As with sets, the order of the elements in a multiset does not matter.
Consider the three element multisets $\{a, b, c\}$ where $abc = 2024$ and a, b and c are positive integers. How many such multisets are there?

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Problem 7.

Last Name

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Answer

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Last Name

ID

Answer

Problem 7. There is a secret set S consisting of positive integers. You only know that S is finite and non-empty, and satisfies the property that whenever $m, n \in S$, it follows that

$$\frac{m+n}{\gcd(m,n)} \in S.$$

What is the smallest possible number of elements in S?

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Problem 8.

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Problem 8. Alice finished printing her thesis. It has between 100 and 1,000 pages. To number the pages, starting with 1, she needed twice as many digits as there are pages. How many pages does her thesis have?

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Problem 9.

Last Name

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Answer

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Answer

Problem 9. Consider a non-zero polynomial $P(x)$ with real coefficients that satisfies

$$P^2(x) - P^2(y) = P(x+y)P(x-y)$$

for all real numbers x and y. If $P(1) = 6$, what is $P(7)$?

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Problem 10.

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Answer

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Answer

Problem 10. Consider the variables $x_1, x_2, \ldots, x_{2024}$. Determine the number of integer solutions to the equation

$$x_1^{2024} + ... + x_{2024}^{2024} = 20242024,$$

where exactly half the x_i are odd.

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