Final Exam Review – Integrals

OpenStax sections: 4.10, 5.1–5.7, 6.1.

Exercises

1. Evaluate the integral:

   (a) $\displaystyle\int_1^4 \frac{3t^2 - \sqrt{t}}{t}\,dt$

   (b) $\displaystyle\int_{-1}^1 (3 - 6x^5)\,dx$

   (c) $\displaystyle\int_0^1 (1-x)^5\,dx$

   (d) $\displaystyle\int_0^1 \sin(2\pi x)\,dx$

   (e) $\displaystyle\int \frac{x^3}{x^4+1}\,dx$

   (f) $\displaystyle\int \sin x \cos(\cos x)\,dx$

   (g) $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

   (h) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$

2. If f is continuous and $\displaystyle\int_0^4 f(x)\,dx = 6$, find $\displaystyle\int_0^2 f(2x)\,dx$.

3. Find $f(\theta)$ if $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 1$, and $f'(0) = 2$.

4. Without computing, determine which of the integrals $\displaystyle\int_1^4 \sqrt{x}\,dx$ or $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}\,dx$ has the larger value. Justify your answer.

5. Given f, sketch the graph of its antiderivative F that passes through the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

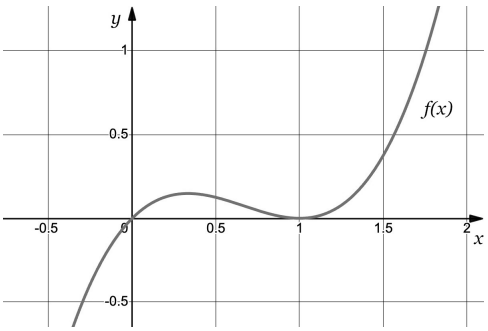

Figure 1: Graph of $f(x)$ used to sketch an antiderivative F.

6. Evaluate:

   (a) $\dfrac{d}{dx}\displaystyle\int_0^1 e^{x^3}\,dx$

   (b) $\dfrac{d}{dx}\displaystyle\int_e^x e^{t^3}\,dt$

   (c) $\displaystyle\int_0^1 \frac{d}{dx}e^{x^3}\,dx$

   (d) $\dfrac{d}{dx}\displaystyle\int_\pi^{x^2} e^{t^3}\,dt$

7. A miniature Christmas train moves back and forth along a straight track in a holiday display. Its velocity is $v(t)$ (in feet per second) and its acceleration is $a(t)$ (in feet per second squared), measured at time t seconds.

   (a) What does the integral $\displaystyle\int_{10}^{30} v(t)\,dt$ represent in the context of the train's motion?

   (b) What does the integral $\displaystyle\int_{10}^{30} |v(t)|\,dt$ represent?

   (c) What does the integral $\displaystyle\int_{10}^{30} a(t)\,dt$ represent?

8. Consider $\displaystyle\int_0^3 (2x+3)\,dx$.

   (a) Estimate the above integral using Riemann sum with three subintervals, taking the sample points to be left endpoints.

   (b) Write the above integral as a limit of Riemann sums, taking the sample points to be right endpoints and evaluate the sum.

   (c) Check your answer using the Fundamental Theorem of Calculus.

9. Determine the value of the integral $\int_1^2 (4f(x)-2x)\,dx$ given that $\int_{-1}^0 f(x)\,dx = 5$, $\int_{-1}^1 f(x)\,dx = 10$, and $\int_0^2 f(x)\,dx = 24$.

10. Use the graph of a function f below to answer the following questions:

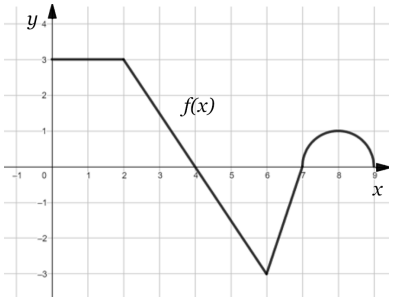

Figure 2: Graph of $f(x)$ used for area questions on $[0, 9]$.

(a) Evaluate $\int_0^9 f(x)\,dx$.

(b) Find the area (not net area) bounded by the graph of $f(x)$ and the x-axis on $[0, 9]$.

(c) If $F(t) = \int_0^t f(x)\,dx$, find $F'(1)$.

11. Find the area of the region bounded by:

(a) $y = \frac{1}{x}, \quad y = x^2, \quad y = 0, \quad x = e.$

(b) $x + y = 0, \quad x = y^2 + 3y.$