

Final Exam Review – Limits

OpenStax sections: 2.1–2.4, 3.1, 3.2, 4.6, 5.2.

Exercises

1. Consider the graph of $f(x)$ below. Compute the following quantities when possible. When a quantity does not exist, explain why.

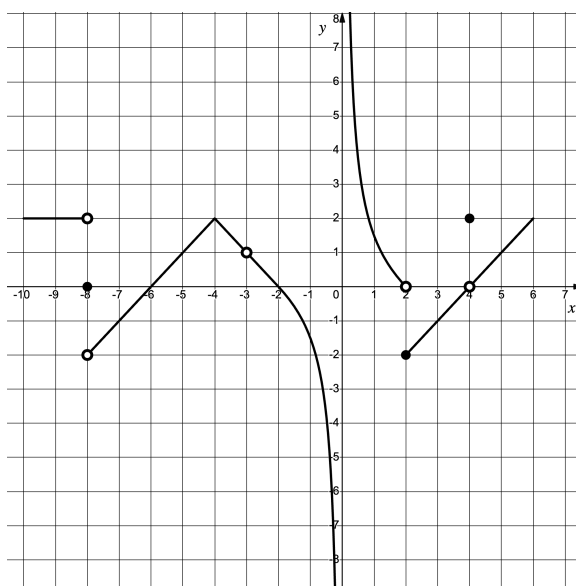


Figure 1: Graph of $f(x)$ used in Exercise 1.

- (a) $\lim_{x \rightarrow 4} f(x)$
- (b) $\lim_{x \rightarrow 2^+} f(x)$
- (c) $\lim_{x \rightarrow -3^-} \frac{f(x) - 1}{x + 3}$
- (d) $\lim_{x \rightarrow 5} \frac{f(x) - f(5)}{x - 5}$
- (e) $\lim_{x \rightarrow -8} f(x)$
- (f) $\lim_{x \rightarrow 0^+} f(x)$
- (g) Average rate of change of $f(x)$ on $[-4, 4]$.

- (h) Instantaneous rate of change of $f(x)$ at $x = -5$.
- (i) Instantaneous rate of change of $f(x)$ at $x = 4$.
- (j) For what values of x is $f(x)$ discontinuous? Classify each discontinuity as removable, jump, or infinite using limits.

2. Evaluate the following limits.

- (a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 2}$
- (b) $\lim_{x \rightarrow 2^+} \sqrt{x + 7} - 2x$
- (c) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
- (d) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 8}$
- (e) $\lim_{x \rightarrow \pi^-} \ln(\sin x)$
- (f) $\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x - 2}$
- (g) $\lim_{x \rightarrow -\infty} \frac{x^2 - 1}{x^5 - 2x^2 + 1}$
- (h) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 4x + 1} - x$
- (i) $\lim_{x \rightarrow 1} \frac{1}{x - 1} - \frac{1}{x^2 + 4x - 5}$
- (j) $\lim_{u \rightarrow 1} \frac{\sin(u - 1)}{2u - 2}$
- (k) $\lim_{x \rightarrow -\infty} \frac{e^x + 1}{1 - x}$
- (l) $\lim_{x \rightarrow \infty} \frac{x + 1}{e^x + 1}$
- (m) $\lim_{x \rightarrow 0^+} x^2 \ln x$
- (n) $\lim_{x \rightarrow \infty} \sqrt{x^{\frac{1}{x}}}$

3. Let

$$f(x) = \begin{cases} ax^2 - 3x & \text{if } x \leq 1, \\ bx + 4 & \text{if } x > 1. \end{cases}$$

For what value or values of a and b , if any, is $f(x)$ differentiable everywhere?

4. Find all values of a such that f is continuous on \mathbb{R} :

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq a, \\ x^2 & \text{if } x > a. \end{cases}$$

5. What type of discontinuity do the following functions have? Justify your response using limits.

(a) $y = \frac{2}{1 - 3x}$

(b) $y = \frac{x^2 - 4}{x - 2}$

6. Does $y = \frac{\sqrt{x^2 + 4}}{1 + 3x}$ have any vertical or horizontal asymptotes? Justify your response using limits.

7. Recognize this limit in order to compute it:

$$\lim_{z \rightarrow 0} \frac{e^{\sin(\frac{\pi}{4} + z)} - e^{\frac{\sqrt{2}}{2}}}{z}.$$

8. Write the integral

$$\int_1^3 (2x - 1) dx$$

as the limit of a Riemann sum, taking the sample points to be the right endpoints of the subintervals and using $a = 1$ and $b = 3$.

9. Compute

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_1^{1+h} \sqrt{1 + e^{t^2}} dt.$$