Final Exam Review – Derivatives

OpenStax sections: 3.1–3.9.

Exercises

1. Using the limit definition, no other method, compute:

   (a) $\dfrac{dy}{dx}$ for $y = \dfrac{1}{2x}$

   (b) $y'(2)$ for $y = \sqrt{x-1}$

2. The graph of $f(x)$ is shown below. On the same graph, with a different color, make a rough sketch of f'.

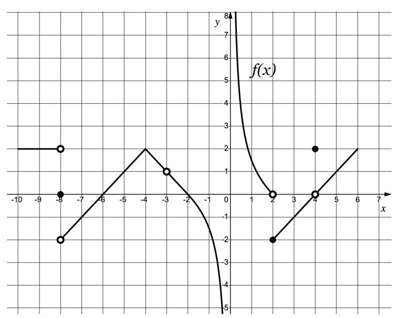

Figure 1: Graph of $f(x)$ used for sketching $f'(x)$.

3. The functions g and h have the following values and derivatives:

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
1	-1	3	0	2

   Using the table above, compute $f'(1)$ for each of the following:

   (a) $f(x) = h(\sqrt{x})$

   (b) $f(x) = \dfrac{g(x)}{h(x) + g(x)}$

   (c) $f = e^x g(x)$

4. Find y':

(a) $y = (x^2+1)^{1785}$

(b) $y = \dfrac{\sqrt{x} + x^2 + 3}{x}$

(c) $y = x \arcsin(x)$

(d) $y = \ln(\sec x)$

(e) $y = \sin^2(\cos(\sin \pi x))$

(f) $xe^y = y \sin x$

(g) $y = 4^{x \ln x}$

(h) $y = \tan\left(\dfrac{x}{1+x^2}\right)$

(i) $y = (\cos x)^x$

(j) $y = \dfrac{\sqrt{x+1}(2+x)^3}{(1+x^2)^{10}}$

5. The graph of $f'(x)$ is shown below. On the same graph, with a different color, sketch the graph of f, given that f is continuous on $[0, 3]$ and $f(0) = 0$.

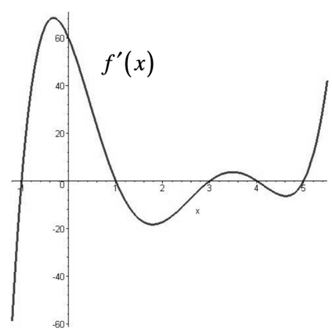

Figure 2: Graph of $f'(x)$ used for sketching $f(x)$.

6. Find $y^{(1785)}\left(\frac{\pi}{3}\right)$, the one thousand seven hundred eighty-fifth derivative of y at $x = \frac{\pi}{3}$, of $y = \cos(x)$.

7. Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 13$ at $(2, 1)$.

8. Let $y = x^3 + 2x - 8$. Find the slope of the tangent line to the inverse function y^{-1} at the point $(4, 2)$, i.e. $\left.\dfrac{d}{dx} y^{-1}(x)\right|_{x=4}$.

9. Two points on the graph of

$$f(x) = x^3 - 3x^2 + 3x + 1785$$

are separated horizontally by 1 unit and have parallel tangents. What is the largest x-coordinate of the two points?