Final Exam Review – Applications

OpenStax sections: various sections.

Exercises

1. Find the local and absolute extrema of $y = \dfrac{3x-4}{1+x^2}$ over $[-2, 2]$.
2. Consider the function $f(x) = x^2 e^{-x}$.
   (a) Does $f(x)$ have an absolute maximum or absolute minimum value ?
   (b) At what values of x (if any) does the function attain a local maximum and a local minimum?
3. Use linearization to approximate $\dfrac{1}{2.03}$.
4. Given that $f(1) = 10$ and $f'(x) \geq 2$ on $[1, 5]$, what is the smallest value that $f(5)$ can have?
5. Verify that the equation $x^3 - 15x + 1 = 0$ cannot possess more than one solution on $[-1, 1]$.
6. Suppose g is a differentiable function satisfying $g(1) = 1$ and $g'(1) = 3$. Let $f(x) = \dfrac{g(x)}{x-2}$. Determine $f'(1)$.
7. Consider the function $y = \dfrac{x}{1-x^2}$. Study it (i.e. state its domain, find $x-$ and $y-$ intercepts, see if it possesses any symmetry, find vertical and horizontal asymptotes, compute first and second derivatives and find critical as well as potential inflection points, organize the latest findings in a table like this:

Interval :		
Sign of f'		
f increasing/decreasing		
Sign of f''		
f concave up/down		
Shape		

and clearly indicate the intervals of increase and decrease, local extrema, intervals of concavity as well as inflection points, then use all this information to sketch the graph).

8. Idem for $y = x\sqrt{x+2}$ and $f(x) = \dfrac{x^2}{x^2+3}$.

9. A 50 ft ladder is placed against a large building. The base of the ladder slips at the rate of 3 ft per minute. Find the rate of change of the height of the top of the ladder above the ground at the instant when the base of the ladder is 30 ft from the base of the building.

10. The altitude of a triangle is increasing at a rate of 1 cm/min, while the area of the triangle is increasing at a rate of $2\,\text{cm}^2$/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is $100\,\text{cm}^2$.

11. If $1200\,\text{cm}^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

12. What are the dimensions of the largest-area rectangle that can be inscribed in a semicircle of radius 3?

13. The graph of the **derivative** of a function $f(x)$ is given below.

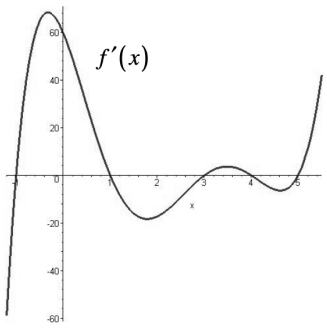

Figure 1: Graph of $f'(x)$ used in the applications exercise.

   (a) For which intervals is the function f increasing or decreasing?

   (b) At which values of x does f attain a local maximum or minimum?

   (c) Sketch a possible graph of f.

14. For what values of the constants a and b is $(1, 3)$ a point of inflection of the curve $y = ax^3 + bx^2$.

15. Find a point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.