

Kossack
Spring 2015

1. Suppose that $\lim_{x \rightarrow a} f(x) = b$ and $\lim_{x \rightarrow a} g(x) = c$. Find $\lim_{x \rightarrow a} f(x)g(x)$.
2. A tangent line is drawn to the hyperbola $xy = c$ at the point $(a, c/a)$. Show that the area bounded by the coordinate axes and the tangent line is independent of a and find this area.
3. Show that at each point where the curves $y = ax^3$ and $x^2 + 3y^2 = b$ intersect, their tangent lines are perpendicular.
4. For what values of c does the equation $\ln x = cx^2$ have exactly one solution?
5. Sketch the graph of $y = x - \tan^{-1}(x)$. Show that the graph has 2 slant asymptotes and find them.
6. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$.
7. A cone is inscribed in a larger cone of height 1, so that its vertex is at the center of the larger cone. Show that the inner cone has maximum volume when its height is $1/3$.
8. If $x \sin(\pi x) = \int_0^{x^2} f(t) dt$, where $f(x)$ is continuous, find $f(4)$.