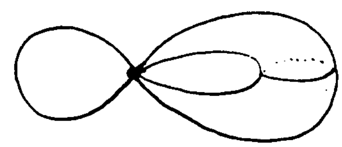
Topology Prelim Fall 2000

X always denotes a topological Hausdorff space, $\mathbb{S}^n$ the n-sphere, and $\mathbb{RP}^2$ the real projective plane. Do any 6 problems.

1. Prove if X is compact and $A \subset X$ is closed then X/A is compact and Hausdorff.

2. Use the Urysohn Theorem to prove that if X is normal then X embedds into a product of closed unit intervals.

3. Prove if X is connected and locally path connected then X is path connected.

4. Let X be the one point union of $\mathbb{S}^1$ and a pinched torus: $X =$ 
   a) Compute $\pi_1(X)$.
   b) Compute $H_*(X)$.

5. a) Prove any map $\mathbb{RP}^2 \longrightarrow \mathbb{S}^1 \times \mathbb{S}^1$ is null homotopic.
   b) Prove there exists a map $\mathbb{S}^1 \times \mathbb{S}^1 \longrightarrow \mathbb{RP}^2$ which is not null homotopic.

6. Compute (up to homomorphism) all connected 2-fold coverings of the one point union of $\mathbb{S}^1$ and $\mathbb{RP}^2$.

7. Prove if $\mathbb{S}^2 \xrightarrow{f} \mathbb{S}^2$ is a map without fixed points then there exists $x \in \mathbb{S}^2$ with $f(x) = -x$.

8. Find (up to homomorphism) all connected closed (ie. compact, without boundary, possibly non-orientable) 2-manifolds M for which every map $M \longrightarrow M$ has a fixed point. (This includes proving your answer is correct.)