Top

Topology Prelim Sept., 1993

(1) Prove that a continuous bijection from a compact space to a Hausdorff space is a homeomorphism.

(2) (a) Prove that an arbitrary product of connected spaces is connected.

(b) Give an example of a connected space which is not path connected.

(3) Prove that if $f : X \to Y$ is a quotient map and X is normal then Y is normal.

(4) Use the homotopy covering property to prove that there exists a well defined bijection $\pi_1(S^1) \to \mathbb{Z}$. ($S^1 =$ 1-sphere)

(5) Let X = real projective 2-space.

(a) Compute $\pi_1(X)$.

(b) Find all coverings of X.

(c) Does there exist a map $f : S^1 \to X$ which does not lift to the universal cover at X? Prove your answer.

(6) Let X be the one point union of $S^1 \times S^1$ and S^1. (S^1 =1-sphere)

(a) Compute $\pi_1(X)$.

(b) Compute $H_*(X)$.

(7) Prove $S^n = \partial B^{n+1}$ is not a retract of B^{n+1}. ($S^n = n$-sphere in $\mathbb{R}^{n+1}$, $B^n = n$-ball in $\mathbb{R}^n$)

(8) Prove any map $f : RP^2 \to RP^2$ has a fixed point. (RP^2 = real projective 2-space)

(9) Let $f : S^2 \to S^2$ be the restriction of the composition of a rotation of $\mathbb{R}^3$ about the z-axis through an angle of $2\pi/3$ radians followed by a reflection in the x-y plane. Define $X = B^3/\sim$ where $x \sim f(x)$ for $x \in S^2 = \partial B^3$. (B^3 = 3-ball in $\mathbb{R}^3$, S^2 = 2-sphere in $\mathbb{R}^3$)

(a) Find a C. W. decomposition of X.

(b) Compute the cellular chain complex of X including the boundary operator.

(c) Compute $H_*(X)$.

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