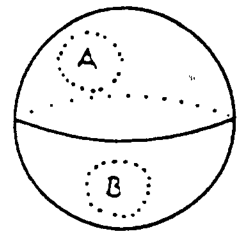
Top

Topology Prelim
September 24, 1992

1) Let A and B be disjoint small round *open* disks in S^2 and let h be a homeomorphism from A to B. Let $X = S^2/\sim$ have the quotient topology where $a \sim h(a)$ for all $a \in A$.



   a) Is X Hausdorff?

   b) Is X connected?

2) Let X be the union of a torus and a disk as shown below. Compute the homology of X.

$X =$ 

3) Let T be a torus minus a small round open disk. Let $\alpha = \partial T$ and use covering spaces to show that $[\alpha] \neq 1 \in \pi_1(T, *)$.

4) State the homotopy lifting theorem for covering spaces and then use it to compute $\pi_1(S^1, *)$.

5) Let X be an arbitrary subset of $\mathbb{R}$. Let $f : X \to \mathbb{R}$ be proper (this means f^{-1} (compact subset of $\mathbb{R}$) = compact subset of X). Show that the graph of f is a closed subset of $\mathbb{R}^2$.

6) Let $f : X \to X$ be continuous.
   a) If $X = S^1 \vee S^1$ must f have a fixed point?
   b) If $X =$  (the wedge product of two closed intervals along an interior point of each) must f have a fixed point?

7) Let $S^1 = \{\vec{v} \in \mathbb{R}^2 \mid |\vec{v}| = 1\}$. Suppose $f : S^1 \to \mathbb{R}^2 - 0$ is continuous and $\vec{v} \cdot f(\vec{v}) > 0$ for all $\vec{v} \in S^1$. Prove that f does not extend to a function from D^2 into $\mathbb{R}^2 - 0$.

8) Let A be the union of two disjoint circles contained in $\mathbb{R}^3 \subset \mathbb{R}^3 \cup \{\infty\} = S^3$ as indicated. Compute the homology of $\mathbb{R}^3 - A$.