Analysis Qualifying Exam: Complex Analysis

August, 2004

Instructions: 5 problems, counted 10 points apiece.

#1. Evaluate the following integral (with $a > 0$) by the method of residues.

$$\int_0^\infty \frac{\cos x}{x^2 + a^2}\, dx$$

#2. Compute the Laurent series for $\dfrac{1}{z^2(1-z)}$ on the annulus $0 < |z| < 1$ and on the annulus $1 < |z| < \infty$.

#3. Find a conformal mapping from $Z = \{z \in \mathbb{C} : Re(z) > 0 \text{ and } Im(z) > 0\}$ onto $W = \{w \in \mathbb{C} : 0 < |w - i| < 2\}$. Include a definition of *conformal mapping*, and briefly indicate why your proposed mapping is indeed conformal. .

#4. Consider the function $f(z) = 1/z$ on the domain $\Omega = \mathbb{C} - \{0\}$.

a) Prove that it is not possible to uniformly approximate $f(z)$ on compact subsets of Ω by polynomials in z. (In other words, show that there is some compact set K in Ω for which there does not exist a sequence of polynomials converging uniformly to f on K.)

b) Indicate how on any closed disk in Ω, $f(z)$ can be uniformly approximated by polynomials.

#5. State and prove the Fundamental Theorem of Algebra by methods of complex analysis. (If you apply Liouville's theorem in your proof, sketch the proof of Liouville's theorem.)