

MATH 2250 Final Exam : Student Guide

UGA Department of Mathematics

Fall 2024

Key Information

- Date: [December 5th, 2024; 7pm](#)¹
- Location: Not your usual classroom (ask your instructor)
- Bring your student photo ID
- The *only* calculator you can use is the **TI30XS - Multiview**. No other models, brands or versions will be accepted!

Justifying Work

For each problem, present your work clearly and show all steps leading to the answer (unless instructed otherwise). Cite any theorems used, such as: the relationship of continuity to differentiability, Extreme Value Theorem (Closed Interval Method), first/second derivative test for local extrema, concavity criteria, L'Hôpital's Rule, Fundamental Theorems of Calculus (Part 1 and 2).

Penalties

Improper use of the following may result in a 20% deduction:

- Missing $+C$ or improper usage of $+C$.
- Improper use of limit notation.
- Improper use of integral or sigma notation.
- Improper equality chains (e.g., $y = x^3 = 3x^2$).
- Algebraic mistakes (e.g., missing parentheses, incorrect variable names).

⁰**Version: Fall 2024.** Initial version approved by the 2250 Instructors on 3/23/2018; revised 2/7/19 Adapted to meet Fall 2020 instructional situation by Mo Hendon, Michael Klipper, and Maury LeBlanc on 7/9/2020. Slightly modified (added integration by substitution) by Jennifer Royal on 12/21/2020, again on 7/6/2021 to bring back some topics removed during COVID teaching. Updated 8/16/21 by Peter Lambert-Cole, Tsung-Wei Hu, Gary Iliev, Woden Kusner, and Jennifer Royal. Updated by Enka Lakuriqi, Phil Bergonio and Lisa Townsley.

¹If you meet the requirements of the university policy or have an emergency (with supporting documentation), there will be a makeup exam for MATH 2250 . Please contact your instructor to arrange to take the makeup exam.

Remarks

- Simplification: exact solutions such as

$$3^{\frac{3}{2}}, e^{-2}, 10^{-4}$$

are acceptable.

Solutions such as

$$\ln 1, e^0, 2^0, \sin\left(\frac{\pi}{4}\right), \cos \pi, \sin^{-1}(1), e^{-\ln 3}, \ln(e^5), \frac{31}{5}, \dots$$

are **not acceptable** and should be simplified. In particular, trig functions of special angles should be evaluated, e.g. $\sec^{-1}(\sqrt{2})$ should be rewritten as $\frac{\pi}{4}$.

- Do not forget to include units (e.g. $\$/ft^2$, m^3 , mi/h , ...) in your final answer
- Problems may combine multiple topics or techniques.
- Only the **TI-30XS Multiview** calculator is allowed.
- The cover page of this exam is attached to this document. Read the instructions carefully.
- No smart watches or devices are permitted during the exam.

Do not forget...

- Properties of logarithmic and exponential functions
- Other precalculus-level formulas (in addition to those listed below)
- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Properties of similar triangles.
- All trig functions which are to be differentiated are to be assumed to have angles in radians as arguments.
- Pythagorean Theorem: $a^2 + b^2 = c^2$.
- Basic trigonometric identities (right triangle identities: $\sin(\theta) = \frac{\text{OPP}}{\text{HYP}}$, ...; quotient identities: $\sec(x) = \frac{1}{\cos x}$, $\tan(x) = \frac{\sin(x)}{\cos(x)}$, ...)
- Trigonometric identity: $\sin^2(x) + \cos^2(x) = 1$.
- Triangle Area $A = \frac{1}{2}bh$ and perimeter.
- Rectangle Area $A = lw$ and perimeter $P = 2l + 2w$.
- Circle area $A = \pi r^2$ and circumference $C = 2\pi r$.
- Cylinder volume $V = \pi r^2 h$ and area $2\pi r^2 + 2\pi r h$ (includes base and lid).
- Sphere volume $V = \frac{4}{3}\pi r^3$ and area $S = 4\pi r^2$.
- Volume circular cone $V = \frac{1}{3}\pi r^2 h$.

Topics List

Limits and Continuity

- The definition of continuity at $x = a$
- Find the limits and one-sided limits of functions (even if not continuous), both analytically and graphically.
- Find limits that approach infinity or have an infinite limit.
- Determine horizontal asymptotes and vertical asymptotes of a function; justify your answer using one or more limits.
- Use L'Hôpital's Rule to find limits (and identify and state the appropriate indeterminate forms that allow you to do so).
- Verify continuity (analytically and graphically).
- Determine intervals on which a function is continuous.
- "Repair" a removable discontinuity by (re)defining the function at that x -value.
- Determine the value of a parameter that makes a piecewise function continuous where the two pieces meet.

Derivatives

- The (limit) definition of the derivative of $f(x)$
- Find the derivative $f'(x)$ from the limit definition of the derivative.
- Use rules to find the derivative; know all rules from back of book through inverse trig function (excluding hyperbolic, parametric, $\operatorname{arcsec}(x)$, $\operatorname{arccot}(x)$, or $\operatorname{arccsc}(x)$).
- Implicit differentiation.
- Compute derivatives at specific points using limited information (e.g., a table, a graph, ...).
- Find an equation of the tangent line at a point.
- Interpret the slope of a function.
- Logarithmic differentiation.
- Properties of even and odd functions. (the derivative of an even function is odd and vice-versa)
- Use differentiation of the appropriate inverse function to verify the differentiation rule for $\ln(x)$.
- Use differentiation of the appropriate inverse function to verify the differentiation rule for $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$ (including an appropriate right triangle diagram or a Pythagorean identity).
- Know how to determine and find:
 - Average rate of change/secant slope, average velocity.

- Instantaneous rate of change/tangent slope.
- Tangent lines and linearization of a function at a point.
- Domain.
- Critical points, inflection points.
- Increasing, decreasing, concave up, concave down regions.
- Local (relative) extrema and absolute (global) extrema.

Applications of derivatives

- Applications involving a tangent line.
- Linearization and differentials.
- Position, displacement, velocity, acceleration problems.
- Interpret the derivative as a rate of change in various contexts.
- Related rates problems.
- Curve sketching from derivative information (including limits, asymptotes, etc.).
- Determine all extrema of a function on a closed interval.
- Applied optimization (open and/or closed interval); **justify** that you have a (relative/absolute) maximum or minimum - finding critical points is not sufficient.

Integration

- Find antiderivatives and solve initial value problems.
- Understand the definite integral as net area.
- Approximate integrals using (left/right) Riemann sums
- Identifying integrals as limits of Riemann sums
- Fundamental Theorem of Calculus (versions 1 and 2)
- Net Change Formula
- Computing integrals using Riemann sums
- Compute definite integrals:
 - By interpreting as area.
 - By the Evaluation Theorem (FTC 2).
 - By substitution.
- Computing areas between curves using integrals.
- Properties of Even and Odd functions.



PRINTED NAME : _____

STUDENT ID : _____

DATE : ____/____/____

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|-------|
| GRADE |
|-------|

INSTRUCTOR : _____

CLASS TIME : _____

INSTRUCTIONS

| N ^o | SCORE | MAX |
|----------------|-------|-----|
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- The exam lasts 3 hours and it has two parts: the first one consists of Multiple Choice (MC) questions, and the second part of Free Response (FR) ones. You must show work for both parts, unless explicitly told otherwise. An unjustified answer will receive no credit. If you are using a shortcut, explain it.
- Your work must be neat and organized. Circle the answer for MC questions and put a box around the final answer for the FR questions. There is only one correct choice for each MC question.
- Smart devices, including smart watches and cell phones, are prohibited and must not be within reach.
- If you plan to use a calculator, only TI-30XS MultiView (the name must match exactly) is permitted; no other calculators or sharing of calculators is allowed.
- Provide an exact answer for each problem. Answers containing symbolic expressions such as $\cos(3)$ and $\ln(2)$ are perfectly acceptable, but $\sin(\pi/2) = 1$.
- If additional space is needed, use the last two pages. Write “cont’d” (continued) in the designated area and continue on the scrap paper by first writing the problem number and then continuing your solution. Work outside the specified area without any indication, will not be graded.