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UNIVERSITY OF GEORGIA

DEPARTMENT OF MATHEMATICS
MATH 2250 - FINAL EXAM
SPRING 2024

PRINTED NAME : Solutions
STUDENT ID : ____________
DATE : ____/____/____

GRADE ____ / 150

INSTRUCTOR : ____________
CLASS TIME : ____________

INSTRUCTIONS

№	SCORE	MAX
1		10
2		5
3		5
4		5
5		5
6		5
7		5
8		5
9		5
10		5
11		5
12		5
13		5
14		5
15		10
16		10
17		5
18		10
19		10
20		5
21		10
22		15
TOTAL		150

- The exam lasts 3 hours and it has two parts: the first one consists of Multiple Choice (MC) questions, and the second part of Free Response (FR) ones. You must show work for both parts. An unjustified answer will receive no credit. If you are using a shortcut, explain it.
- Your work must be neat and organized. Circle the answer for MC questions and put a box around the final answer for the FR questions. There is only one correct answer for each MC question.
- Smart devices, including smart watches and cell phones, are prohibited and must not be within reach.
- If you plan to use a calculator, only TI-30XS MultiView (the name must match exactly) is permitted; no other calculators or sharing of calculators is allowed.
- Provide an exact answer for each problem. Answers containing symbolic expressions such as cos(3) and ln(2) are perfectly acceptable.
- If additional space is needed, use the last two pages. Write ‘ctd’ (continued) in the designated area and continue on the scrap paper by first writing the problem number and then continuing your solution. Work outside the specified area without any indication, will not be graded.

Part I: Multiple Choice

Show work and circle the final answer.

1. [10 pts] Evaluate the following limits:

______ 5 PTS (a) $\lim_{x \to 1} \frac{x^2 - 3}{2 - x}$ [form: $\frac{-2}{1}$] $= -2$

Ⓐ 0 Ⓒ -2 Ⓔ DNE

Ⓑ -1 Ⓓ ∞ Ⓕ Something else

______ 5 PTS (b) $\lim_{x \to \infty} \frac{4x^2 - 10}{1 + 2x - 9x^2}$ [form: $\frac{\infty}{\infty}$] $= \lim_{x \to \infty} \frac{4x^2}{-9x^2} = -\frac{4}{9}$

Ⓐ $-\frac{4}{9}$ Ⓒ 2 Ⓔ ∞

Ⓑ $\frac{4}{9}$ Ⓓ 4 Ⓕ DNE

_____ PTS 2. [5 pts] Determine all horizontal asymptotes of $g(x) = \frac{\sqrt{9x^2+1}}{2x}$.

(A) $y = -\frac{9}{2}$

(B) $y = -\frac{3}{2}$

(C) $y = \frac{3}{2}$

(D) $y = \frac{9}{2}$

(E) $y = -\frac{3}{2}$ and $y = \frac{3}{2}$

(F) $y = -\frac{9}{2}$ and $y = \frac{9}{2}$

(G) $y = 0$

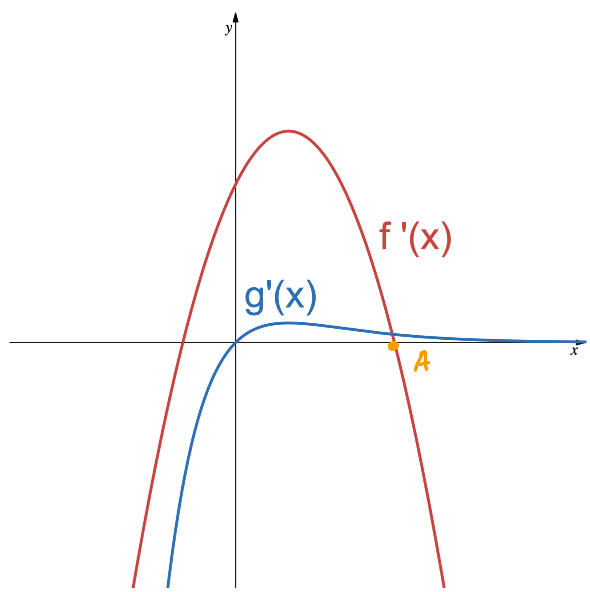
(H) No horizontal asymptotes

$$\lim_{x\to\infty} \frac{\sqrt{9x^2+1}}{2x} = \lim_{x\to\infty} \frac{3x}{2x} = \frac{3}{2}$$

$$\lim_{x\to-\infty} \frac{\sqrt{9x^2+1}}{2x} = \lim_{x\to-\infty} \frac{-3x}{2x} = -\frac{3}{2}$$

$\therefore$ $y = -\frac{3}{2}$ and $y = \frac{3}{2}$ are the two horizontal asymptotes of $f(x)$.

_____ PTS 3. [5 pts] Below are the graphs of the **derivatives** of two functions $f(x)$ and $g(x)$. Carefully read the following statements and determine which of them must necessarily be true.



Statement (I): $f(x)$ has a local minimum.
Statement (II): $g(x)$ has a root.
Statement (III): $g(x)$ is increasing everywhere.

(A) I only

(B) II only

(C) I and II

(D) I and III

(E) I, II, and III

I: $f(x)$ has a local minimum $\Leftrightarrow$ $f(x)=0$ and $f'(x)$ changes sign from +ve to -ve: this is the case at A.

II: $g(x)$ has a root : nothing to do with first derivative.

III: $g(x)$ is increasing everywhere $\Leftrightarrow$ $g'(x)$ is above the x-axis. Not the case.

_____ PTS 4. [5 pts] Determine $\dfrac{dy}{dx}$ if $x^4 = x^2 y - y^3$.

(A) $\dfrac{dy}{dx} = \dfrac{2x}{1-3y^2}$

(B) $\dfrac{dy}{dx} = \dfrac{4x^3}{2x-3y^2}$

(C) $\dfrac{dy}{dx} = \dfrac{4x^3-2xy}{x^2-3y^2}$

(D) $\dfrac{dy}{dx} = \dfrac{4x^3-x^2y}{2x-3y^2}$

(E) $\dfrac{dy}{dx} = \dfrac{4x^3(x^2-y^2)-x^4(2x-2y)}{(x^2-y^2)^2}$

$$x^4 = x^2 y - y^3$$

$\frac{d}{dx}$

$$4x^3 = 2xy + x^2 y' - 3y^2 y'$$

$$y' = \frac{4x^3 - 2xy}{x^2 - 3y^2} \;//$$

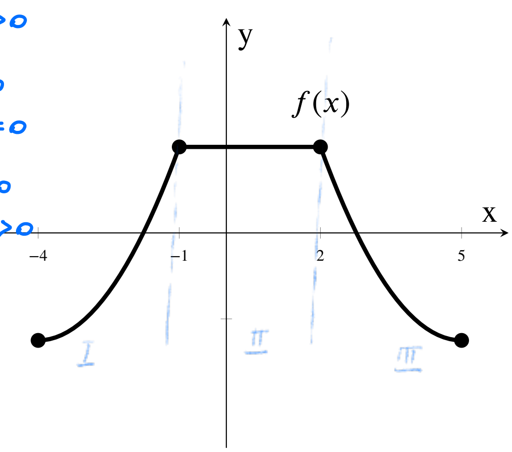
_____ PTS 5. [5 pts] Consider the graph of a function $f(x)$ which is twice differentiable except at a few points on the interval $[-4, 5]$:

I: $f(x)$ - increasing $\Leftrightarrow f'(x) > 0$
$f(x)$-concave up $\Leftrightarrow f''(x) > 0$

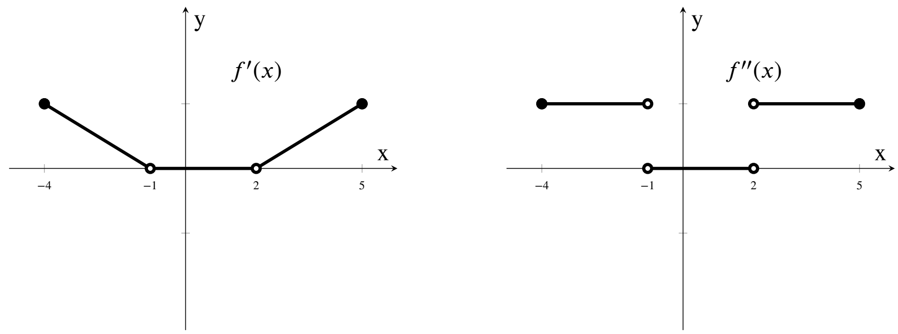
II: $f(x)$ - constant $\Leftrightarrow f'(x) = 0$
$f(x)$ - constant $\Leftrightarrow f''(x) = 0$

III: $f(x)$ -decreasing $\Leftrightarrow f'(x) < 0$
$f(x)$-concave up $\Leftrightarrow f''(x) > 0$

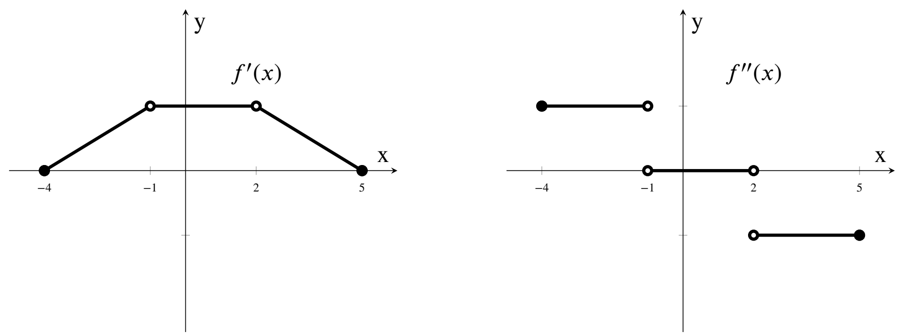
(D) encompasses the above conclusions.



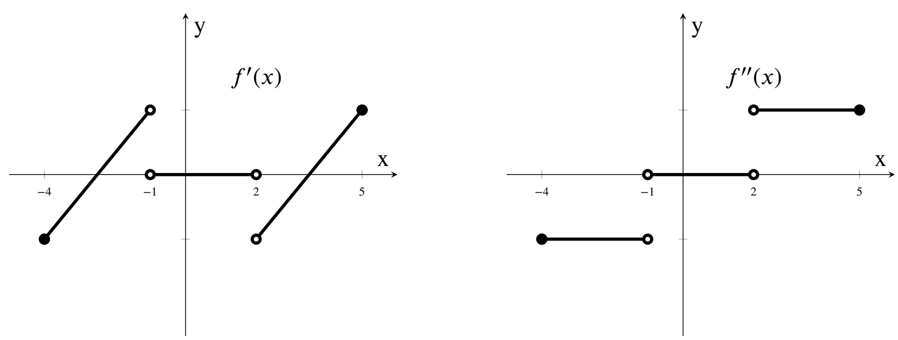
Which of the following sketches **best describes** the graphs of $f'(x)$ and $f''(x)$?

A


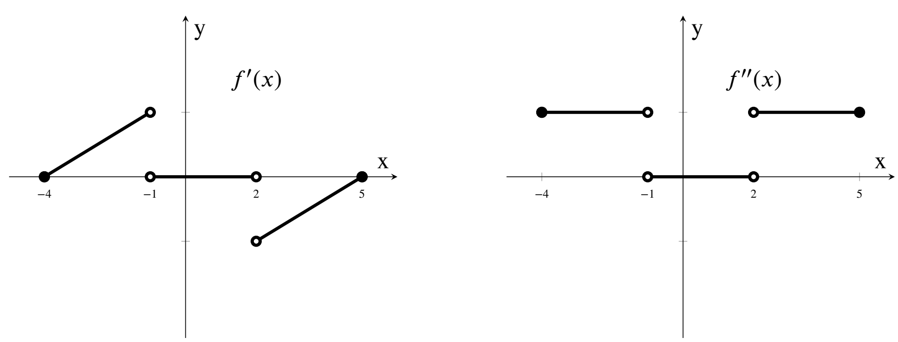
y
$f'(x)$
x
−4 −1 2 5
y
$f''(x)$
x
−4 −1 2 5

B


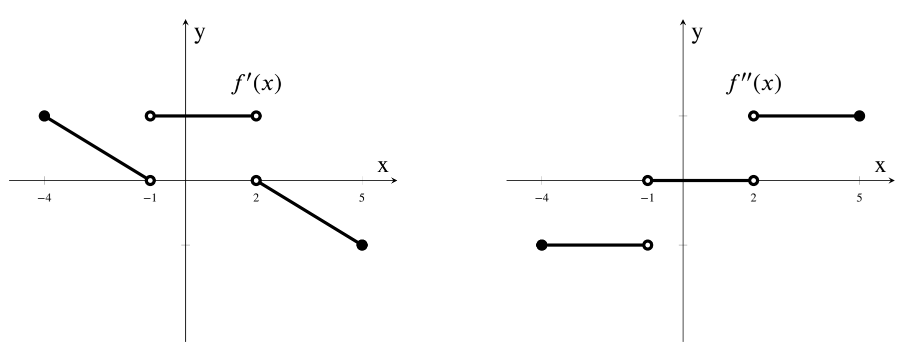
y
$f'(x)$
x
−4 −1 2 5
y
$f''(x)$
x
−4 −1 2 5

C


y
$f'(x)$
x
−4 −1 2 5
y
$f''(x)$
x
−4 −1 2 5

D


y
$f'(x)$
x
−4 −1 2 5
y
$f''(x)$
x
−4 −1 2 5

E


y
$f'(x)$
x
−4 −1 2 5
y
$f''(x)$
x
−4 −1 2 5

______ PTS 6. [5 pts] At how many values of x on the interval $[-1,2]$ does the function $f(x) = \frac{1}{4}x^4 - x^3 + x^2 + 3$ attain **absolute minimum**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$f'(x) = x^3 - 3x^2 + 2x$

Critical values: $x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x(x-1)(x-2) = 0$

$\Rightarrow x = 0, 1, 2 \in [-1, 2]$

Checking the function values at critical points as well as endpoints:

$f(-1) = \frac{1}{4} + 1 + 1 + 3 = \frac{21}{4}$

$f(0) = 3$

$f(1) = \frac{1}{4} - 1 + 1 + 3 = \frac{13}{4}$

$f(2) = 4 - 8 + 4 + 3 = 3$

absolute minimum at $x = 0$ and $x = 2$.

______ PTS 7. [5 pts] If $f(x) = \int_{\sqrt{x}}^{2} \sin(t^2)\,dt$ then $f'(x)$ is

(A) $-\frac{\sin(x)}{\sqrt{x}}$ (D) $-\frac{\sin(x)}{2\sqrt{x}}$ (G) $-\frac{\sin(x)}{2x}$

(B) $-\frac{\sin(x)}{x}$ (E) $-\frac{\sin(\sqrt{x})}{2\sqrt{x}}$ (H) $\sin(x)$

(C) $\frac{\sin(x)}{x}$ (F) $\frac{2\sin(x)}{\sqrt{x}}$ (I) $\sin(4) - \sin(x)$

$$f'(x) = \frac{d}{dx}\int_{\sqrt{x}}^{2} \sin(t^2)\,dt$$

$$= \frac{d}{dx}\left(-\int_{2}^{\sqrt{x}} \sin(t^2)\,dt\right)$$

$$= -\sin((\sqrt{x})^2)\cdot\frac{1}{2\sqrt{x}}$$

$$= -\frac{\sin x}{2\sqrt{x}} \quad //$$

_____ PTS 8. [5 pts] Approximate the area under the graph $f(x) = 1 + \sin^2(\pi x)$ on $\left[0, \frac{1}{2}\right]$ using a right-endpoint Riemann sum with **three subintervals** of equal length.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{3}{4}$ (E) $\frac{2}{5}$

$0 \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2}$

$$\Delta x = \frac{\frac{1}{2} - 0}{3} = \frac{1}{6}$$

$$A \approx \frac{1}{6}\left[f\left(\frac{1}{6}\right) + f\left(\frac{1}{3}\right) + f\left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{6}\left[1 + \sin^2\left(\frac{\pi}{6}\right) + 1 + \sin^2\left(\frac{\pi}{3}\right) + 1 + \sin^2\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{6}\left[1 + \left(\frac{1}{2}\right)^2 + 1 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1 + 1^2\right]$$

$$= \frac{1}{6}\left[4 + \frac{1}{4} + \frac{3}{4}\right]$$

$$= \frac{5}{6}$$

_____ PTS 9. [5 pts] The rate of growth of rabbit population in a forest is given by $G(t) = 1000e^{0.05t}$, where $G(t)$ is measured in number of rabbits per year, and t is the time in years since the start of 2010. Using the correct units, what does $\int_2^7 G(t)\,dt$ represent?

(A) The integral represents the area under the curve of $G(t)$ between the years 2012 and 2017, measured in square rabbits.

(B) The total increase in the number of rabbits between the years 2012 and 2017, measured in rabbits.

(C) The rate of growth of the rabbit population at year 2017, measured in rabbits per year.

(D) The total number of rabbits born since 2012, measured in rabbits.

(E) The average rate of growth of the rabbit population between the years 2012 and 2017, measured in rabbits per year.

______ PTS 10. [5 pts] Which of the following is the derivative of the function

$$f(x) = \frac{e^{\arctan(x)}}{1+x^2}\,?$$

$$f'(x) = \frac{(1+x^2)\cdot\frac{1}{1+x^2}\,e^{\arctan x} - 2x\,e^{\arctan x}}{(1+x^2)^2}$$

$$= \frac{e^{\arctan x}(1-2x)}{(1+x^2)^2}$$

(A) $f'(x) = \dfrac{e^{\arctan(x)}}{2x}$

(B) $f'(x) = \dfrac{e^{\arctan(x)}(1-2x)}{(1+x^2)^2}$

(C) $f'(x) = \dfrac{e^{\arctan(x)}}{(1+x^2)^2}\left(1 - \dfrac{2x}{(1+x^2)^2}\right)$

(D) $f'(x) = e^{\arctan(x)}\left(\dfrac{1}{1+x^2} - 2x\right)$

(E) $f'(x) = \dfrac{e^{\arctan(x)}(1+x^2-2x)}{(1+x^2)^2}$

______ PTS 11. [5 pts] Which of the following is true about the function

$$H(x) = \frac{x^2-5x+6}{x^2-8x+15}\,?$$

$$= \frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x-5)} = \frac{x-2}{x-5} \quad \text{for } x \neq 3$$

(A) $H(x)$ has a vertical asymptote at $x = 3$, but has a removable discontinuity at $x = 5$.

(B) $H(x)$ has vertical asymptotes at $x = 3$ and $x = 5$, but has a removable discontinuity at $x = 2$.

(C) $H(x)$ has a vertical asymptote at $x = 5$, but has a removable discontinuity at $x = 3$.

(D) $H(x)$ has vertical asymptotes at $x = 2$ and $x = 3$, but has a removable discontinuity at $x = 5$.

(E) $H(x)$ has a vertical asymptote at $x = 2$ and $x = 5$, but has a removable discontinuity at $x = 3$.

______ PTS 12. [5 pts] Classify all local extrema of $\frac{[\ln(x)]^2}{x}$.

(A) $x = 1$ is a local minimum and $x = e^2$ is a local maximum

(B) $x = 1$ is a local minimum and $x = 0$ and $x = e^2$ are local maxima

(C) $x = 1$ is a local maximum and $x = e^2$ is a local minimum

(D) $x = 1$ is the only extrema and is a local minimum

(E) $x = e^2$ is the only local extrema and is a local maximum

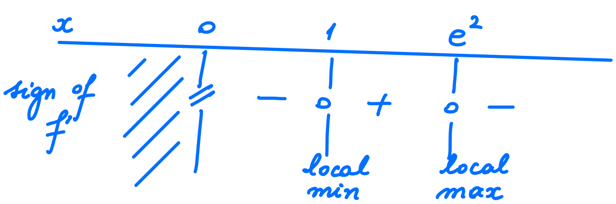
dom $f(x) = (0, \infty)$

$$f'(x) = \frac{\cancel{x} \cdot 2 \ln x \cdot \frac{1}{\cancel{x}} - (\ln x)^2}{x^2}$$

$$= \frac{\ln x\,(2 - \ln x)}{x^2}$$

$= 0$: $\ln x = 0 \Rightarrow x = 1$ or $2 - \ln x = 0 \Rightarrow x = e^2$

DNE: $x = 0$



______ PTS 13. [5 pts] Let F and G be two differentiable functions satisfying

$$F\left(\frac{\pi}{4}\right) = -1, \quad F'\left(\frac{\pi}{4}\right) = 3, \quad G\left(\frac{\sqrt{2}}{2}\right) = 2, \text{ and } G'\left(\frac{\sqrt{2}}{2}\right) = 5.$$

Which of the following is the value of the derivative of $F(x) \cdot G(\cos(x))$ at $x = \pi/4$?

(A) $6 + \frac{5}{\sqrt{2}}$ (B) $6 - \frac{5}{\sqrt{2}}$ (C) $5 + \frac{6}{\sqrt{2}}$ (D) $5 - \frac{6}{\sqrt{2}}$ (E) None of the above

$$\frac{d}{dx}\left(F(x) \cdot G(\cos x)\right)\Big|_{x=\frac{\pi}{4}} = F'(x)\, G(\cos x) + F(x)\, G'(\cos x)(-\sin x)\Big|_{x=\frac{\pi}{4}}$$

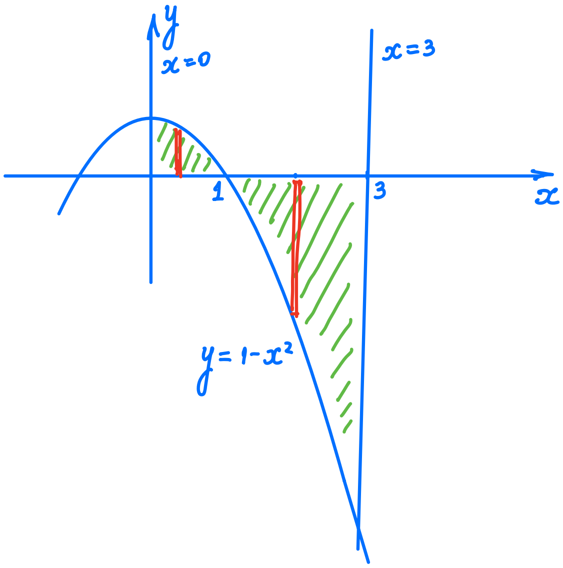
$$= \underbrace{F'\left(\tfrac{\pi}{4}\right)}_{=3} G\Big(\underbrace{\cos \tfrac{\pi}{4}}_{=\frac{\sqrt{2}}{2}}\Big) + F\left(\tfrac{\pi}{4}\right) G'\Big(\underbrace{\cos \tfrac{\pi}{4}}_{=\frac{\sqrt{2}}{2}}\Big)\Big(-\underbrace{\sin \tfrac{\pi}{4}}_{=\frac{\sqrt{2}}{2}}\Big)$$

$$= 3 \cdot \underbrace{G\left(\tfrac{\sqrt{2}}{2}\right)}_{=2} + (-1)\, \underbrace{G'\left(\tfrac{\sqrt{2}}{2}\right)}_{5}\left(-\tfrac{\sqrt{2}}{2}\right)$$

$$= 3 \cdot 2 + \frac{5\sqrt{2}}{2} = 6 + \frac{5\sqrt{2}}{2} \,//$$

_____ PTS 14. [5 pts] Find the area enclosed by the curve $y = 1 - x^2$, the x-axis, and the lines $x = 0$, and $x = 3$.

(A) 6 (B) -6 (C) 7 (D) $\frac{22}{3}$ (E) $\frac{16}{3}$



$$A = \int_0^1 (1-x^2)\,dx + \left|\int_1^3 (1-x^2)\,dx\right|$$

or

$$= \int_0^1 (1-x^2)\,dx + \int_1^3 (x^2-1)\,dx$$

$$= \left(x - \frac{x^3}{3}\right)\Big|_0^1 + \left(\frac{x^3}{3} - x\right)\Big|_1^3$$

$$= \left[\left(1-\frac{1}{3}\right) - 0\right] + \left[(9-3) - \left(\frac{1}{3}-1\right)\right]$$

$$= \frac{2}{3} + 6 + \frac{2}{3}$$

$$= \frac{22}{3} \quad //$$

Part II: Free Response

Present all work leading to your final answer clearly and in a structured manner. Show all relevant steps and justify them.

15. [10 pts] Compute the following limits:

_____ 5 PTS (a) $\lim_{x\to\frac{\pi}{2}} \dfrac{x\sin(2x)}{\cos(x)}$

$\left[\text{form } \frac{0}{0}\right] \underset{\text{L'Hôpital}}{=} \lim_{x\to\frac{\pi}{2}} \dfrac{\sin(2x) + 2x\cos(2x)}{-\sin x}$

$\left[\text{form } \frac{-\pi}{-1}\right] = \pi$ //

OR use the double angle formula

$$\lim_{x\to\frac{\pi}{2}} \frac{x\sin(2x)}{\cos x} = \lim_{x\to\frac{\pi}{2}} \frac{x\, 2\sin x\, \cancel{\cos x}}{\cancel{\cos x}} = \frac{\pi}{2}\cdot 2\cdot \sin\frac{\pi}{2} = \pi \ //$$

_____ 5 PTS (b) $\lim_{x\to 0} x\sin\left(\dfrac{1}{x}\right)$

$-1 < \sin\left(\frac{1}{x}\right) < 1$

$-x < x\sin\left(\frac{1}{x}\right) < x$

$x\to 0^+$: $-x \to 0$, $x \to 0$

$\therefore$ by Squeeze Theorem.

$\lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right) = 0$

Same thing as $x\to 0^-$.

OR $t = \frac{1}{x}$, as $x\to 0$ $t\to\pm\infty$

$\lim_{t\to\pm\infty} \frac{\sin t}{t} = 0$ as $\frac{-1}{t} < \frac{\sin t}{t} < \frac{1}{t}$

$t\to+\infty$: $\frac{-1}{t}\to 0$, $\frac{1}{t}\to 0$

$\therefore$ by Squeeze Theorem $\frac{\sin t}{t} \xrightarrow[t\to\infty]{} 0$

Idem for $t\to-\infty$.

______ PTS 16. [10 pts] Use the limit definition of the derivative to compute the derivative of $f(x) = x^3$ at $x = 2$. **No credit will be awarded for any other method.**

[Hint: recall that $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$*]*

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} + 12h + 6h^2 + h^3 - \cancel{8}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12 + 6h + h^2)}{\cancel{h}}$$

$$= 12 \,//$$

17. [5 pts] Consider the function $\ln(x)$.

______ 3 PTS (a) Find its linearization at $x = 1$.

$$L(x) = f(1) + f'(1)(x-1)$$

$$= \ln(1) + \left.\frac{1}{x}\right|_{x=1} (x-1)$$

$$= x - 1 \,//$$

______ 2 PTS (b) Use the above linearization to estimate $\ln(1.02)$.

$$\ln(1.2) \approx 1.2 - 1 = 0.2 \,//$$

18. [10 pts] Evaluate the following indefinite integrals:

_____ 5 PTS (a) $\displaystyle\int \frac{x+1}{2x}\,dx$

$= \frac{1}{2}\int \left(1 + \frac{1}{x}\right) dx$

$= \frac{1}{2}\left(x - c_1 + \ln|x| + c_2\right)$

$= \boxed{\frac{1}{2}x + \frac{1}{2}\ln|x| + C}$ where $C = \frac{1}{2}c_1 + \frac{1}{2}c_2$.

_____ 5 PTS (b) $\displaystyle\int \sqrt{2x+4}\,dx$

let $u = 2x+4$

$du = 2dx$

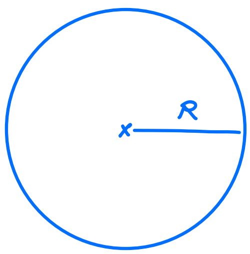
$= \int \sqrt{u}\,\frac{du}{2}$

$= \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{1}{3}u^{3/2} + C$

$= \frac{1}{3}(2x+4)^{3/2} + C$ //

______ PTS 19. [10 pts] In a petri dish, a colony of bacteria takes on a circular shape. The radius of this colony is increasing steadily at a rate of 0.2 millimeters per second. What's the rate at which the area of the circle is increasing at the instant when the circumference reaches 20π millimeters? Make sure to include the units in your final answer.



Data:

$\frac{dR}{dt} = 0.2 \ \frac{mm}{s}$

$\frac{dA}{dt} = ?$ when $C = 20\pi$ mm

Relation: $A = \pi R^2$

$$\frac{dA}{dt} = \underbrace{2\pi R}_{= C} \frac{dR}{dt}$$

$$= 20\pi \cdot 0.2$$

$$= 4\pi$$

Answer: The area of the circle is increasing at a rate of $\boxed{4\pi \ mm^2/s}$ when C reaches 20π mm.

______ PTS 20. [5 pts] Determine the derivative of $f(x) = [\cos(2x)]^{3x}$. *[You might want to use logarithmic differentiation.]*

Using logarithmic differentiation: $\ln f(x) = \ln([\cos(2x)]^{3x})$

$$= 3x \ln(\cos(2x))$$

$$\frac{f'(x)}{f(x)} = 3x \cdot \frac{-2 \sin(2x)}{\cos(2x)} + 3 \ln(\cos 2x)$$

$= \tan(3x)$

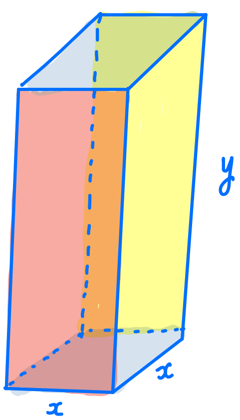
Thus, $\boxed{f'(x) = (-6x \tan(2x) + 3 \ln(\cos 2x))[\cos(2x)]^{3x}}$

_____ PTS 21. [10 pts] You have been put in charge of designing a box for a revolutionary product in your favorite company. The product manager tells you that the **rectangular** box needs to have volume 200 cm^3, it should have a square base, and it should be painted in such a way that:

- The top and bottom faces are squares, painted blue. (blue paint costs \$1/cm^2)
- The front face is painted red (red paint costs \$2/cm^2), while the rear face is painted yellow (yellow paint costs \$0.5/cm^2).
- The left and the right faces are not painted.

The aim is to minimize the cost of painting the box.

_____ 2 PTS (a) Draw a picture of the box. Mention explicitly which variable you use to denote which side, especially which variable you use to denote a side of the square base.



_____ 3 PTS (b) Write all the constraints and equations relevant to the situation.

Constraint: $x^2 y = 200$ [cm^3]

Cost: $C(x) = 2(x^2)\cdot 1 + xy\cdot 2 + xy \cdot 0.5$ [\$/cm^2]

$= 2x^2 + \frac{5}{2}xy$ [\$/cm^2]

______ 2 PTS (c) Write a formula of the function you want to optimize in terms of a single variable, and give a reasonable domain for the function (this is the domain where the problem makes sense).

$$C(x) = 2x^2 + \frac{5}{\cancel{2}} x \frac{\overset{100}{\cancel{200}}}{x^2}$$

$$= 2x^2 + \frac{500}{x}, \qquad x \in (0, \infty)$$

______ 3 PTS (d) Minimize the function using calculus, and thus determine the dimensions of the box that minimize the cost of painting.

$$C'(x) = 4x - \frac{500}{x^2}$$

$$4x - \frac{500}{x^2} = 0$$

$$4x^3 = 500$$

$$x^3 = \frac{500}{4} = 125$$

$$\Rightarrow x = 5 \quad \text{and} \quad y = 8$$

Now we need to verify that $x = 5$ indeed represents the optimal dimension for the base that minimizes the cost of painting.

	(0	$x=5$	∞)
sign of C'	$-$	0	$+$

This table shows that $x = 5$ cm ($y = 8$ cm) are the dimensions that yield the lowest cost of painting the box.

22. [15 pts] Consider the function

$$f(x) = \frac{x^4}{2} + 2x^3 .$$

Here is some useful information: the roots of $f(x)$ are -4 and 0, its domain is the set of all real numbers and its derivative is

$$f'(x) = 2x^3 + 6x^2 .$$

______ 2 PTS (a) Find the critical points of $f(x)$.

$$f'(x) = 2x^3 + 6x^2 = 0$$

$$x^2(2x+6) = 0 \Rightarrow x = 0, -3$$

______ 5 PTS (b) Find $f''(x)$ and determine where the curve is concave down.

$$f''(x) = 6x^2 + 12x$$

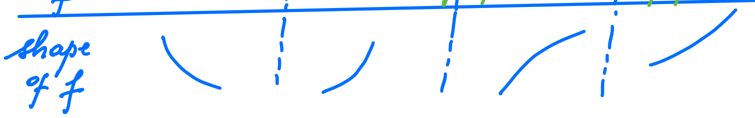
$$6x(x+2) = 0 \Rightarrow x = 0, -2$$

x		-2		0	
sign of f''	$+$	0	$-$	0	$+$

The curve is concave down on $(-2, 0)$.

______ 5 PTS (c) Complete a number line/table/chart, partitioning it appropriately and indicating the sign of f', the sign of f'' and the shape of f (increasing/decreasing and concave up/down).

x		-3		-2		0	
sign of f'	$-$	0 loc min	$+$		$+$	0	$+$
sign of f''	$+$		$+$	0 infl. pt.	$-$	0 infl. pt.	$+$
shape of f							



______ 3 PTS (d) Sketch the graph of $f(x)$.