Departmental Syllabus for MATH 2260: Calculus II for Science and Engineering

Course description (from the UGA Bulletin): Volumes, arclength, work, separable differential equations. Techniques of integration. Sequences and series, convergence tests, power series and Taylor series. Vectors in three-dimensional space, dot product, cross product, lines and planes.

Textbook: Currently, most sections of MATH 2260 use *OpenStax Calculus* (freely available at https://openstax.org/details/books/calculus-volume-2); the course corresponds to most of Volume 2 plus Chapter 2 of Volume 3. *University Calculus* by Hass, Heil, Weir, Bogacki, and Thomas (currently on its fourth edition, ISBN 0134995546) is also an option. References to sections in both books below are current as of April 2023.
For online homework, WebAssign (paid) offers assignments based on OpenStax, and there are also many problems on all Calculus II subjects in WeBWorK (free), including ones connected to sections in a previous edition of *University Calculus*.

Outline: MATH 2260 begins with a quick review of the integration that was covered at the end of 2250, and proceeds to four largely independent units. Suggested durations below are based on a four class-period-per-week schedule, with eight days left for review and tests. If time is short, subjects of lower priority might include numerical integration and alternating series.

Here are the topics covered in the course, with associated recommended learning outcomes:

0. **Review of integration (OpenStax v.2 Chapter 1; HHWBT Chapter 5; 4 days)**
   • Estimate areas (both total and signed) under curves using Riemann Sums with left endpoints, right endpoints, or midpoints.
   • Use the Fundamental Theorem of Calculus, Part 2, to evaluate definite integrals (including those requiring u-substitution, handling limits of integration appropriately).
   • Apply basic integration formulas.

1. **Applications of definite integrals (13 days)**
   a) Areas between curves (OSv2 2.1; HHWBT 5.6; 1 day)
   • Determine the area of a region bounded by two curves, by integrating with respect to the independent variable, or by integrating with respect to the dependent variable.

   b) Volumes using cross-sections (OSv2 2.2; HHWBT 6.1 and start of Section 6.2; 3.5 days)
   • Find the volume of a solid by integrating the area of its cross-sections
   • Use the disk method or the washer method, as appropriate, to find the volume of a solid of revolution.

c) Volumes using cylindrical shells (OSv2 2.3; HHWBT 6.2; 1.5 days)

• Find the volume of a solid of revolution using the method of cylindrical shells.

• Decide whether the shell method, the disk method, or the washer method is better suited to a given problem.

d) Arc length and areas of surfaces of revolution (OSv2 2.4; HHWBT 6.3 and 6.4; 2 days)

• Determine the length of a curve between two points, for curves given by equations y=f(x) or x=g(y).

• Find the surface area of a solid of revolution

e) Physical applications including work (OSv2 2.5; HHWBT 6.5; 2 days)

• Calculate the work done:

- by a variable force acting along a line;
- in pumping liquid from one height to another

f) Exponential change and separable differential equations (OSv2 4.3; HHWBT 7.2; 3 days)

• Use separation of variables to solve a differential equation.

• Model and solve problems involving heating and cooling or exponential growth or decay using separable differential equations.

2. Techniques of integration (13 days)

a) Integration by parts (OSv2 3.1; HHWBT 8.1; 2 days)

• Use integration by parts to find antiderivatives:

- in one step;
- in multiple steps;
- in "wrap-around" problems, e.g. with $e^x \cos x$

b) Trigonometric integrals (OSv2 3.2; HHWBT 8.2; 2 days)

• Solve integration problems involving:

- products of powers of sine and cosine
- products of powers of secant and tangent
- using one or more trigonometric identities before integrating

c) Trigonometric substitution (OSv2 3.3; HHWBT 8.3 2 days)

• Use trigonometric substitution to solve integration problems involving the square root of a sum or difference of two squares.

d) Partial fractions (OSv2 3.4; HHWBT 8.4; 2 days)

• Integrate rational functions in which the degree of the numerator is less than the

degree of the denominator using partial fractions, including cases involving non-repeated linear factors, repeated linear factors, and quadratic factors.
• Use polynomial long division to reduce a general rational function to one that can be integrated with partial fractions.

e) General practice with/synthesis of integration techniques (1 day)
• Decide which technique or combination of techniques is well-suited to a given integration problem.

f) Numerical integration (OSv2 3.6; HHWBT 8.6; 2 days)
• Apply the trapezoidal rule or Simpson's rule to estimate the value of an integral.
• Use error bounds to determine the number of steps needed in a numerical integration technique to estimate a given definite integral to a given accuracy.

g) Improper integrals (OSv2 3.7; HHWBT 8.7; 2 days)
• Give examples of improper integrals that converge or diverge.
• Evaluate improper integrals:
- over infinite intervals;
- over finite intervals when the integrand has an infinite discontinuity within the interval.

3. **Infinite sequences and series (15 days)**

a) Sequences (OSv2 5.1; HHWBT 9.1; 2 days)
• Determine the convergence or divergence of a given sequence.
• Calculate the limit of a sequence (if it exists).

b) Infinite series including the divergence (nth term) test (OSv2 5.2 and start of 5.3; HHWBT 9.2; 2 days)
• Determine the partial sums of a series.
• Determine when a geometric series or a telescoping series converges or diverges, and compute its sum if it converges.
• Identify, when applicable, that a series diverges using the nth term test.

c) The integral test (OSv2 5.3; HHWBT 9.3; 1 day)
• Use the integral test to determine the convergence/divergence of a series.
• Use the p-series test to determine the convergence/divergence of a series.

d) Comparison tests (OSv2 5.4; HHWBT 9.4; 1 day)
• Use the (direct) comparison test or the limit comparison test to determine the convergence/divergence of a series.

e) The ratio and root tests (OSv2 5.6; HHWBT 9.5; 1 day)

• Use the ratio test or the root test to determine the (absolute) convergence of a series.

f) Alternating series, absolute vs. conditional convergence (OSv2 5.5; HHWBT 9.6; 1 day)

• Test an alternating series for convergence using the alternating series test.

• Give examples of series which converge absolutely and which converge conditionally.

g) Power series (OSv2 6.1 and 6.2; HHWBT 9.7; 2 days)

• Determine the radius of convergence and interval of convergence of a power series.

h) Taylor and Maclaurin series (OSv2 6.3; HHWBT 9.8 and 9.9; 3 days)

• Determine the first *n* terms of a Taylor (or Maclaurin) series.

• Estimate the error of the Taylor polynomial approximation of a given order to a given function.

i) The binomial series and applications of Taylor series (OSv2 6.4; HHWBT 9.10; 2 days)

• Recognize Taylor series expansions of common functions.

• Use Taylor series to solve differential equations.

• Use Taylor series to evaluate non-elementary integrals.

4. **Vectors and the geometry of space (7 days)**

a) Vectors in two and three dimensions (OSv3 2.1 and 2.2; HHWBT 11.1 and 11.2; 2 days)

• Perform, and correctly interpret, basic vector operations in two and three dimensions: addition, scalar multiplication, and magnitude.

b) The dot product (OSv3 2.3; HHWBT 11.3; 1.5 days)

• Compute the dot product between two vectors.

• Use the dot product to find the angle between two given vectors.

• Find the orthogonal projection of one vector along another.

c) The cross product (OSv3 2,4; HHWBT 11.4; 1.5 days)

• Compute the cross product between two three-dimensional vectors.

• Given two three-dimensional vectors, find a third one that is orthogonal to both.

d) Lines and planes in space (OSv3 2.5; HHWBT 11.5; 2 days)

• Give a vector (or a parametric) equation of a line through a given point and in a given direction, or through two given points.

• Find an equation of a plane passing through three given points.

(approved by the Curriculum Committee, April 2023)