MATH 3000 SYLLABUS

Spring 2010

At the moment we have not settled on a universal textbook, although for a number of years several instructors have used the text by Shifrin/Adams. Below are listed the “standard" topics that every course should cover, along with section numbers in that text and an approximate time allotment (under a MWF teaching schedule).

MATH 3000, as the numbering suggests, is one of our “transitional" or “bridge" courses, in which students must be learning to come to terms with definitions, abstract concepts, and **proof writing**, *along with* computational skills. Homework and tests—not just the lectures—should contain a significant percentage of proofs. (For example, we should strive for every C student to be able to check whether some subset of $\mathbb{R}^n$ is a subspace, every B student should be able use the definition of linear independence to prove that if $\mathbf{v}$ and $\mathbf{w}$ are linearly independent vectors, then so are $\mathbf{v}+\mathbf{w}$ and $2\mathbf{v}-\mathbf{w}$.)

There should be some discussion of linear transformations and of the interplay between linear algebra and geometry (of particular importance for the mathematics education students), e.g., rotations and reflections, and the interpretation of determinant as signed volume. Hopefully, no matter the text, students should come to understand that a subspace can be specified in two ways:

(i) implicitly: cutting it out by equations (e.g., $\mathbf{N}(A)$ for an appropriate matrix A);
(ii) parametrically: giving a basis for it (e.g., $\mathbf{C}(B)$ for an appropriate matrix B).

Students should understand completely how to go back and forth between these, as well as the relevance of the fundamental equations $\mathbf{R}(A)^{\perp} = \mathbf{N}(A)$ and $(V^{\perp})^{\perp} = V$. The course should also include discussion of eigenvalues and eigenvectors, along with some substantive application(s). Try to finish the course with the Spectral Theorem. (If time permits, some discussion of computer graphics or the matrix exponential and systems of ODEs might be fun.)

Here is a list of topics for the course, together with a breakdown by sections in T. Shifrin and M. R. Adams, *Linear Algebra: A Geometric Approach*, second edition.

Required Core Topics	Recommended Supplementary Topics	Sections	Days
Vectors, dot product		1.1–1.2	4
Systems, Gaussian elimination		1.3–1.4	3
Theory of linear systems		1.5	2
	Applications	1.6	2
Matrix algebra, linear maps (treat elementary matrices lightly)		2.1–2.5	6
Vector spaces		3.1–3.4	7
	Abstract vector spaces	3.6	2
Least squares, orthogonal bases		4.1–4.2	3
Change-of-basis formula		4.3	2
	Linear maps on abstract vector spaces	4.4	1
Determinants		5.1–5.2	2.5
	Geometric interpretations	5.3	1
Eigenvalues and eigenvectors		6.1–6.2	3
	Applications	6.3	1.5
	Spectral Theorem	6.4	2
		Total:	42

We reiterate that the topics we've listed as *required core* should all be covered. Different instructors will allot time to various supplementary topics, but we hope that everyone will do at least two or three applications-oriented sections. —M. Adams, E. Azoff, T. Shifrin