Study Guide for Real Analysis Exam

Undergraduate Analysis

Continuity and differentiation in one real variable

Metric spaces and compactness in analysis

Sequences and series

Uniform convergence and uniform continuity

Taylor's theorem

Weierstrass approximation theorem

References: [2] Chapters 2, 3, 4, 5, 7; [1] Sections 0.6.

Measure and Integration

Measures on $\mathbb{R}^n$ and on σ-algebras

Measurable and integrable functions

Convergence theorems: *Fatou's lemma, the monotone and dominated convergence theorems and Egoroff's theorem*

Notions of convergence: *uniform, pointwise, almost everywhere, and in norm*

Fubini and Tonelli theorems

References: [1] Chapters 1, 2; [3] Chapters 1, 2, 6.

Function Spaces

The Banach spaces L^1 and L^∞:

- Completeness
- Convolutions and approximations to the identity
- Linear functionals and realizing L^∞ as the dual of L^1

Hilbert space and L^2 spaces:

- Schwarz inequality and orthogonality
- Linear functionals and the Riesz representation theorem
- Bessel's inequality, orthonormal basis, and Parseval's identity
- Trigonometric series: *trigonometric polymonials are dense in both* $C([0,1])$ *(with respect to the uniform metric) and in* $L^2([0,1])$

References: [1] Sections 5.2, 5.5, 6.2; [3] Chapter 4.

References

[1] G. B. Folland, *Real Analysis*, 2nd edition, John Wiley & Sons, Inc.

[2] W. Rudin, *Principles of Mathematical Analysis*, 3rd edition, Macmillan.

[3] E. M. Stein and R. Shakarchi, *Real Analysis*, Princeton University Press.

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