

# AMERICAN MATHEMATICAL SOCIETY

## *Notices*

*Edited by* GORDON L. WALKER

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# MEETINGS

## CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
564	January 27-29, 1960 (66th Annual Meeting)	Chicago, Illinois	Dec. 14
565	February 19-20, 1960	Tucson, Arizona	Jan. 6
566	February 27, 1960	New York, New York	Jan. 6
567	April 14-16, 1960	New York, New York	Mar. 1
568	April 22-23, 1960	Chicago, Illinois	Mar. 1
569	April 22-23, 1960	Berkeley, California	Mar. 1
570	June 18, 1960	Missoula, Montana	May 5
571	August 29-September 3, 1960 (65th Summer Meeting)	East Lansing, Michigan	July 15
572	October 22, 1960	Worcester, Massachusetts	Sept. 8

\*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

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# FIVE HUNDRED SIXTY-FIRST MEETING

Wake Forest College  
Winston-Salem, North Carolina  
November 20-21, 1959

## PROGRAM

The five hundred sixty-first meeting of the American Mathematical Society will be held at Wake Forest College, Winston-Salem, North Carolina, on November 20-21, 1959 in the Science Building.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor Trevor Evans will address the Society on "Decision problems in algebra" at 2:00 P.M. Friday in Room 14, Science Building.

Sessions for contributed papers will be held from 11:00 A.M. to noon and from 3:30 P.M. to 5:00 P.M. on Friday and from 10:00 A.M. to noon on Saturday. Abstracts of the papers to be presented at these sessions appear on pages 629-635 of these NOTICES.

Registration headquarters will be in the Science Building and will be open from 10:00 A.M. Friday through noon Saturday. Dormitory accommodations will be available for the nights of November 20 and 21 at the rate of \$3.00 per person per night. A banquet will be held Friday at 7:00 P.M., the charge being \$2.25 per person. Banquet and dormitory reservations should be made by November 14 and may be obtained through Professor J. Robert Johnson, Jr., Box 7404, Reynolda Station, Winston-Salem, North Carolina. The Cafeteria and the Magnolia Room in Reynolda Hall will be available for meal service. Reservations for hotel and motel rooms should be made directly with the chosen motel and hotel. Minimum rates are listed below for conveniently located hotels and motels:

	<u>Single</u>	<u>Double</u>
Beacon Hill Motel 3618 Reynolda Road	\$5.00	\$8.00
Blue Bird Motel Highway 52 North	5.00	7.00 - 8.00
Carolina Hotel 407 West 4th	4.00 - 5.00	6.50 - 8.00
Kembly Inn Motel Cloverdale Avenue	6.00	9.50
Robert E. Lee Hotel N. Cherry Street	4.75 - 9.25	8.75 - 12.75
Stevens Hotel (colored) 526 4th Street N. E.	From \$3.00	From \$4.00

Winston-Salem is located on United States Highways 52, 158, 311, and 421. It is served by the Southern Railroad and by the Atlantic Greyhound and Queen City Trailways bus lines. Airline service is provided by Eastern Air Lines, Capital Airlines, and Piedmont Airlines.

Wake Forest College is located approximately four miles northwest of the center of Winston-Salem, and may be reached by following U. S. Highway 421 west until signs marking the college entrance are seen.

Mail and telegrams for those attending the meeting may be sent in care of the Department of Mathematics, Wake Forest College, Reynolda Station, Winston-Salem, North Carolina.

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### PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

FRIDAY, 11:00 A.M.

Session on Algebra and Theory of Numbers, Room 114, Science Building

11:00 - 11:10

- (1) A note on exponential sums

Professor Leonard Carlitz, Duke University (561-5)

11:15 - 11:25

- (2) Note on a number theoretical paper of Sierpinski

Professor A. T. Brauer, University of North Carolina  
(561-7)

11:30 - 11:40

- (3) Irreducibility of certain classes of Legendre polynomials.  
Preliminary report

Mr. R. F. McCoart, University of North Carolina  
(Introduced by Professor A. T. Brauer)

11:45 - 11:55

- (4) On an upper bound for the difference between consecutive primes

Professor C. N. Moore, University of Cincinnati  
(561-14)

FRIDAY, 2:00 P.M.

Invited Address, Room 14, Science Building

Decision problems in algebra (One hour)

Professor Trevor Evans, Emory University

FRIDAY, 3:30 P.M.

General Session, Room 114, Science Building

3:30 - 3:40

- (5) Primitive idempotents in the semigroup of measures  
Professor H. S. Collins, Louisiana State University  
(561-1)

3:45 - 3:55

- (6) On the almost-periodic motion of an earth satellite.  
Preliminary report  
Professor R. A. Struble, North Carolina State College  
(561-11)

4:00 - 4:10

- (7) Notes on sequential multivariate tests on means  
Mr. R. A. Bradley and Mr. J. E. Jackson, Florida State  
University and Eastman Kodak Company (561-16)  
(Introduced by Dr. T. L. Wade)

SATURDAY, 10:00 A.M.

Session on Topology, Room 113, Science Building

10:00 - 10:10

- (8) Countable paracompactness and cb-spaces  
Professor J. G. Horne, Jr., U. S. Naval Ordnance Test  
Station, China Lake, California, and University of  
Georgia (561-2)

10:15 - 10:25

- (9) Homogeneity of the hyperspace of a continuum. Preliminary  
report  
Mr. Jack Segal, University of Georgia (561-4)

10:30 - 10:40

- (10) Some sequences of groups for which an onto inverse  
limit is unique  
Mr. P. D. Hill, Alabama Polytechnic Institute (561-9)

10:45 - 10:55

- (11) Polyhedra in  $S^n$  whose complements have uniformly abelian  
local fundamental groups  
Mr. C. H. Edwards, Jr., University of Tennessee  
(561-12)  
(Introduced by Professor O. G. Harrold)

11:00 - 11:10

- (12) A lifting and neighborhood extension theorem for mappings into fiber spaces

Professor M. K. Fort, Jr., University of Georgia

(561-19)

11:15 - 11:25

- (13) Plane sections of topological spheres

Professor R. D. Anderson, Louisiana State University

(561-20)

SATURDAY, 10:00 A.M.

Session on Analysis, Room 114, Science Building

10:00 - 10:10

- (14) Openness of the derivative of a complex function

Professor R. L. Plunkett, Florida State University

(561-3)

10:15 - 10:25

- (15) A nonlinear boundary problem for harmonic functions

Professor M. H. Martin, University of Maryland (561-6)

10:30 - 10:40

- (16) A note on weakly fundamental sequences in Banach spaces

Professor R. D. McWilliams, Florida State University

(561-10)

10:45 - 10:55

- (17) An extension of an early result of Stieltjes

Professor J. T. Moore, University of Florida (561-13)

11:00 - 11:10

- (18) Operator-moment problems. Preliminary report

Professor J. S. MacNerney, University of North Carolina

(561-15)

11:15 - 11:25

- (19) Eigenfunction expansions for operator algebras

Dr. R. T. Harris, Duke University

11:30 - 11:40

- (20) Summability of derived Fourier series

Dr. B. J. Boyer, Florida State University (561-18)

SUPPLEMENTARY PROGRAM

(To be presented by title)

- (21) Multi-valued functions, inverse limits and groups

Professor R. D. Anderson, Louisiana State University

- (22) Some problems concerning Kummer's congruences for the Euler numbers and polynomials  
Professor Leonard Carlitz, Duke University, and Professor Jack Levine, North Carolina State College
- (23) Discontinuity relations for charged, self-gravitating, compressible relativistic fluids  
Professor Nathaniel Coburn, University of Michigan
- (24) Unitary products of arithmetical functions  
Professor Eckford Cohen, University of Tennessee
- (25) A theorem on orthonormal series  
Professor R. P. Gosselin, University of Connecticut
- (26) Some applications of equi-infinitesimal analysis  
Dr. C. A. Muses, The Barth Foundation
- (27) The analog of Haar's problem for the existence and characterization of best Tchebycheff approximations  
Dr. J. R. Rice, National Bureau of Standards, Washington, D. C. and California Institute of Technology

Gerald B. Huff  
Associate Secretary

Athens, Georgia  
October 15, 1959

## FIVE HUNDRED SIXTY-SECOND MEETING

University of Southern California  
Los Angeles, California  
November 21, 1959

### PROGRAM

The five hundred sixty-second meeting of the American Mathematical Society will be held on Saturday, November 21, 1959, at the University of Southern California, Los Angeles, California. All sessions will meet in Founders Hall, located at the corner of University Avenue and 34th Street.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be an address at 1:30 P.M. in Room 133, Founders Hall, by Professor John Todd of the California Institute of Technology. Professor Todd's subject is "Conformal mapping and computers".

Sessions for contributed papers will be held at 10:00 A.M. and at 3:00 P. M. in Room 133, Founders Hall. Abstracts of the papers to be presented at these sessions appear on pages 636-640 of these NOTICES. There are cross references to the abstracts in the program. For example the title of paper (1) in the program is followed by (562-12) indicating that the abstract can be found under the designation 562-12 among the published abstracts. Late papers can be added to the program. For information concerning late papers, inquire at the registration desk.

Registration for the meeting will begin at 9:00 A.M. The registration desk will be located outside Room 133 in Founders Hall.

Tea will be served on the second floor of the Commons Building following the afternoon sessions for contributed papers. All members attending the meeting are invited to this tea.

Luncheon will be available at the cafeteria in the Commons Building. A section will be reserved for people attending the meeting. The Commons Building is on Childs Way, just west of University Avenue.

The University of Southern California can be reached from downtown Los Angeles via the "J" streetcar on 7th Street or Grand Avenue, or by automobile, taking the Harbor Freeway to Exposition Boulevard. Parking near the campus may be a problem. If there are no spaces available on the street, the Administration Parking Lot at University Avenue and 35th Street can be used for a fee of 50 cents.

There are numerous hotels in downtown Los Angeles. The Coliseum Hotel, located at the corner of Figueroa Street and Santa

Barbara Avenue is conveniently near the U. S. C. campus.

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### PROGRAM OF THE SESSIONS

The time limit for each contributed paper is ten minutes. The contributed papers are scheduled at 15 minute intervals so that listeners can circulate between the different sessions. To maintain this schedule, the time limit will be strictly enforced.

SATURDAY, 10:00 A.M.

General Session, Room 133, Founders Hall

10:00 - 10:10

- (1) Sets of integers defined by approximations  
Professor Ivan Niven, University of Oregon (562-12)

10:15 - 10:25

- (2) On Lie algebras of Killing-Cartan-Seligman type  
Dr. Richard Block, California Institute of Technology  
(562-11)

10:30 - 10:40

- (3) Topologies for countably and finitely generated abelian groups  
Mr. R. E. Haymond, University of California, Riverside  
(562-6)

10:45 - 10:55

- (4) Homomorphisms of commutative Banach algebras  
Professor W. G. Bade, University of California, Berkeley, and Professor P. C. Curtis, Jr., University of California, Los Angeles (562-9)

11:00 - 11:10

- (5) The Wedderburn principal theorem for certain commutative Banach algebras. II  
Professor W. G. Bade, University of California, Berkeley, and Professor P. C. Curtis, Jr., University of California, Los Angeles (562-10)

11:15 - 11:25

- (6) A structure theorem for topological lattices  
Professor L. W. Anderson and Professor L. E. Ward, Jr., University of Oregon (562-2)

11:30 - 11:40

- (7) One-dimensional topological semi-lattices  
Professor L. W. Anderson and Professor L. E. Ward, Jr., University of Oregon (562-3)

11:45 - 11:55

- (8) Continua which converge homeomorphically  
Professor C. E. Burgess, University of Utah (562-8)

SATURDAY 1:30 P.M.

Invited Address, Room 133, Founders Hall

- Conformal mapping and computers (One hour)  
Professor John Todd, California Institute of Technology

SATURDAY, 3:00 P.M.

General Session, Room 133, Founders Hall

3:00 - 3:10

- (9) Arrangement of 24 points on a sphere  
Professor R. M. Robinson, University of California,  
Berkeley (562-5)

3:15 - 3:25

- (10) Monotone networks  
Dr. G. J. Minty, University of Washington (562-4)

3:30 - 3:40

- (11) Analytical formulation of the physical properties of water  
and steam  
Professor M. A. Dengler, Arizona State University  
(562-1)

3:45 - 3:55

- (12) A note on the completeness of order statistics  
Professor C. B. Bell, Jr., University of California,  
La Jolla and San Diego State College, Professor David  
Blackwell, University of California, Berkeley, and Mr.  
Leo Breiman, University of California, Berkeley (562-7)

#### SUPPLEMENTARY PROGRAM

(To be presented by title)

- (13) Abelian groups with minimal systems of generators.  
Preliminary report  
Mr. S. A. Khabbaz, University of Kansas
- (14) Polyhedral sections of convex bodies  
Professor V. L. Klee, Jr., University of Copenhagen and  
University of Washington
- (15) Further applications of equi-infinitesimal analysis. Pre-  
liminary report  
Dr. C. A. Muses, The Barth Foundation

- (16) Generalized Heisenberg-Schroedinger systems and one-dimensional cohomology  
Professor I. E. Segal, University of Chicago
- (17) On analytic sets in topological spaces  
Professor Maurice Sion, University of California,  
Berkeley
- (18) Radial cluster sets and the distribution of values of meromorphic functions  
Dr. W. B. Woolf, University of Washington

R. S. Pierce  
Associate Secretary

Seattle, Washington  
October 12, 1959

## FIVE HUNDRED SIXTY-THIRD MEETING

Wayne State University  
Detroit, Michigan  
November 27-28, 1959

### PROGRAM

The five hundred sixty-third meeting of the American Mathematical Society will be held at Wayne State University, Detroit, Michigan, on Friday and Saturday, November 27-28, 1959. All sessions will be held in the McGregor Memorial Conference Center.

The Committee to Select Hour Speakers for Western Sectional Meetings has invited Professor H. J. Ryser of the Ohio State University and Professor J. H. C. Whitehead of the University of Oxford and the University of Chicago to address the Society. Professor Ryser will speak on "Matrices of zeros and ones" at 2:00 P.M. on Friday, November 27. Professor Whitehead will address the Society on Saturday, November 28 at 10:00 A.M. on the subject "Smooth manifolds". Both lectures will be delivered in the Auditorium of the McGregor Memorial Conference Center.

Sessions for contributed papers will be held on Friday afternoon at 3:15 P.M. and Saturday morning at 11:15 A.M. Abstracts of the papers to be presented at these sessions appear on pages 641-648 of these NOTICES. There are cross references to the abstracts in the program. For example the title of paper (1) in the program is followed by (563-12) indicating that the abstract can be found under the designation 563-12 among the published abstracts. If necessary, there will be a special session on Saturday afternoon for the presentation of contributed papers which failed to meet the deadline.

Members of the Society and their guests are invited to tea on Friday afternoon at 4:30 P.M. in the McGregor Memorial Conference Center.

### REGISTRATION

The registration desk will be located in the lobby of the McGregor Memorial Conference Center which is on Second Avenue at Ferry (southeast of and near the intersection of the Ford and Lodge expressways). Those who attend the meetings are requested to register at any time from noon to 5:00 P.M. on Friday and from 9:30 A.M. to noon Saturday.

### ROOMS

Two hotels located within two blocks of the McGregor Memorial Conference Center have agreed to hold rooms for reservations

ham Spitzbart, University of Wisconsin in Milwaukee, to a professorship.

Wahab, Georgia Institute of Technology, to an associate professorship.

Willoughby, Georgia Institute of Technology, to an associate professorship.

Zelinsky, Northwestern University, to an associate professorship.

The following appointments to instructorships are announced:

Yale University: Mr. Arno Cronheim; University of British Columbia: Mr. Barron Brainerd, Dr. M. D. Marcus; Barnard College, Columbia University: Dr. Jacqueline L. Penez; Cornell University: Dr. L. J. Fetters; University of Delaware: Mr. W. G. Spohn; Franklin and Marshall College: Mr. W. H. Leser; Illinois Institute of Technology: Dr. F. J. C. Ross; Marquette University: Rev. L. J. Heider, Mr. J. E. Kelly; Massachusetts Institute of Technology: Mr. W. C. Fox, Dr. Sigurd H. Nielsen; University of Michigan: Dr. T. R. Jenkins, Mr. J. H. Walter; Mount Holyoke College: Mr. G. F. Feeman; University of Nebraska: Dr. J. H. Duerksen; Ohio State University: Mr. J. S. Minas, Dr. J. M. Shapiro; Oregon State University: Mr. S. S. Spector; Portland State Extension Center, Portland, Oregon: Mr. R. G. Byrne; Princeton University: Mr. F. P. Palermo, Dr. G. J. Janz; University of Rochester: Mr. E. H. Batho; St. Peter's University, New Jersey: Mr. J. W. Toole; Trinity College: Mr. J. H. Duerksen; Tulane University: Mr. Leon Brown, Dr. J. R. Isbell; University of Washington: Dr. H. A. Forrester; Racine Extension, University of Wisconsin: Mr. W. F. Ames.

Dr. C. Briant of the Oak Ridge National Laboratory died on March 5. He had been a member of the Society for seventeen years.

Dr. J. P. Smith of Georgetown University died on March 5, 1957, at the age of sixty-six years. He had been a member of the Society for

4:15 - 4:25

- (5) Algebraic topological methods for the synthesis of switching systems. IV. Minimization of singular Boolean trees  
Dr. J. P. Roth, IBM Research Center, Yorktown Heights,  
New York (563-17)

SATURDAY, 10:00 A.M.

Invited Address

Smooth manifolds (One hour)

Professor J. H. C. Whitehead, University of Oxford  
and University of Chicago

SATURDAY, 11:15 A.M.

Session on Topology, McGregor, Room J

11:15 - 11:25

- (6) Isotopies in 3-manifolds with boundaries. I

Dr. J. M. Kister, University of Wisconsin and University of Michigan (563-3)

11:30 - 11:40

- (7) Linearization of mappings

Professor Johannes deGroot, Purdue University and University of Amsterdam (563-5)

11:45 - 11:55

- (8) Clusters of indecomposability

Mr. R. L. Kelley and Professor P. M. Swingle, University of Miami (563-7)

12:00 - 12:10

- (9)  $E^4$  is the Cartesian product of a totally non-Euclidean space and  $E^1$

Mr. R. H. Rosen, University of Michigan (563-1)

12:15 - 12:25

- (10) Complex homogeneous contact manifolds. Final report

Professor W. M. Boothby, Washington University (563-15)

12:30 - 12:40

- (11) Metrizable Köthe spaces

Mr. R. R. Welland, Purdue University (563-8)  
(Introduced by Professor Casper Goffman)

Session on Analysis and Applications, McGregor, Room F

11:15 - 11:25

- (12) Equi-absolute convergence of eigenfunction expansions

Dr. G. W. Hedstrom, University of Michigan (563-10)

11:30 - 11:40

- (13) Strong Riesz summability of orthogonal expansions  
Mr. D. J. Eustice, Purdue University (563-14)

11:45 - 11:55

- (14) Uniform convergence factors of orthonormal expansions  
Professor Daniel Waterman, University of Wisconsin,  
Milwaukee, and Professor S. A. Husain, University of  
Seattle (563-16)

12:00 - 12:10

- (15) A characterization in  $E^3$  of the everywhere regular solu-  
tion of the reduced wave equation  
Professor O. G. Owens, Wayne State University (563-6)

12:15 - 12:25

- (16) Notes on a nonlinear generalization of Laplace's equation  
Professor Philip Cooperman, University of Pittsburgh  
(563-11)

12:30 - 12:40

- (17) Solution of nonlinear algebraic problems  
Mr. D. D. Morrison, Space Technology Laboratories,  
Incorporated, Los Angeles, California (563-4)

#### SUPPLEMENTARY PROGRAM

(To be presented by title)

- (18) Uniform boundedness for groups  
Professor I. L. Glicksberg, University of Notre Dame
- (19) Some special transformation groups  
Professor I. L. Glicksberg, University of Notre Dame
- (20) Variation diminishing convolutions. Preliminary report  
Professor I. I. Hirschman, Jr., Washington University
- (21) Isotopies in 3-manifolds with boundaries. II  
Dr. J. M. Kister, University of Wisconsin and Univer-  
sity of Michigan
- (22) The geometry of equi-infinitesimals  
Dr. C. A. Muses, The Barth Foundation

J. W. T. Youngs  
Associate Secretary

Bloomington, Indiana  
October 9, 1959

# PRELIMINARY ANNOUNCEMENT OF MEETING

## SIXTY-SIXTH ANNUAL MEETING

Chicago, Illinois  
January 27-29, 1960

The sixty-sixth annual meeting of the American Mathematical Society will be held at the Conrad Hilton Hotel in Chicago, Illinois, on January 27, 28 and 29, 1960.

The Presidential Address will be delivered by Professor Richard Brauer of Harvard University on Thursday, January 28, at 1:00 P.M. Professor Julian Schwinger of Harvard University will give the thirty-third Josiah Willard Gibbs lecture, entitled "Quantum field theory," on Wednesday, January 27, at 8:00 P.M.

Two invited addresses are planned, but the Committee to Select Hour Speakers for Annual and Summer Meetings has not yet made a final decision in the matter.

The award of the Cole Prize in Algebra will be made at 2:00 P.M. on January 28, and is to be followed by the Annual Business Meeting. The Council will meet on Wednesday, January 27, at 4:00 P.M., and the Trustees the following morning at 10:00 A.M.

Due to a convention prior to this meeting of the Society, there will be no space in the Conrad Hilton for the night of January 26. The registration desk will first open at noon, and the sessions will begin in the afternoon of January 27. Members are urged to make reservations, beginning with the night of January 27, directly with the Conrad Hilton Hotel, and identify themselves as members. Tentative rates are as follows:

		<u>Lakeside</u>	<u>Lakefront</u>
Single Rooms	\$7.00 to \$ 9.00 up	\$12.00	\$ 13.50 to \$17.00
Double Rooms	12.00 to 15.00	16.00	17.50 to 21.00
Twin Rooms	13.00 to 20.00	16.50 to 19.50	19.00 to 21.00
Suites	23.00 to 25.00	40.00 up	47.00 up

Due to the lack of time available, it will be impossible for the Society to accommodate members whose abstracts do not arrive in the Providence Office prior to the deadline - December 14, 1959.

J. W. T. Youngs  
Associate Secretary

Bloomington, Indiana  
October 7, 1959

## FIVE HUNDRED SIXTY-FIFTH MEETING

University of Arizona  
Tucson, Arizona  
February 18-20, 1960

The five hundred sixty-fifth meeting of the American Mathematical Society will be held at the University of Arizona, Tucson, Arizona, February 18-20, 1960. This extraordinary meeting of the Far Western Section of the Society is planned as a part of the seventy-fifth anniversary celebration of the University of Arizona. A two day Symposium on "Differential Geometry" is scheduled, along with the usual activities of a one day meeting. Arrangements are being made to provide an unusual Western program of entertainment for members and their families who attend this meeting.

The Committee on Arrangements for this meeting would like to know as early as possible the approximate number of people to expect. Since February is the peak of the tourist season in Tucson, there will be great difficulty in securing accommodations unless reservations are made before the meeting. The Department of Mathematics at the University of Arizona will handle requests for hotel, motel, or dormitory rooms, provided they are received early enough. For these reasons, members of the Society who plan to attend this meeting are requested to fill out the reservation form on page 651 of these NOTICES and return it to Professor Harvey Cohn, Department of Mathematics, University of Arizona, Tucson 25, Arizona, NOT LATER THAN January 15, 1960.

Additional information on this meeting will be given in the Preliminary Announcements section of the December issue of these NOTICES.

R. S. Pierce  
Associate Secretary

Seattle, Washington  
October 7, 1959

## **ACTIVITIES OF OTHER ASSOCIATIONS**

A SYMPOSIUM ON COMBINATORIAL PROBLEMS will be held at Princeton University on April 12-13, 1960. The Symposium will be under the chairmanship of Professor A. W. Tucker, and will be devoted to reporting recent results in the field of combinatorial problems, including those of the IBM Combinatorial Problems Institute. The Symposium will be jointly sponsored by the Society for Industrial and Applied Mathematics and IBM Corporation. Further information concerning the Symposium and hotel accommodations may be obtained by writing Professor A. W. Tucker, Box 708, Fine Hall, Princeton University, Princeton, New Jersey.

THE OPERATIONS RESEARCH SOCIETY OF AMERICA will hold their sixteenth national meeting in Pasadena, California at the Huntington-Sheraton Hotel, November 11, 12 and 13, 1959. The banquet speaker will be General Don Coupland, USAF, Deputy Commander for Ballistic Missiles, Headquarters San Bernardino Air Material Area, who will speak on Ballistic Missiles Management. The luncheon speaker will be Dr. George Steiner, Director, Division of Research, Graduate School of Business Administration, UCLA, who will speak on Problems of Mobilization and Civil Defense.

Technical sessions will be held on: Missiles and Space Systems, Military Strategy and Tactics, National Security Objectives, Traffic and Transportation, Evaluation Through Experimentation, Business Management and Planning, Production Planning, Medical Applications, Simulation of Operations, Mathematical Models, Education for Operations Research, Reliability and Military and Commercial Aircraft Operations.

In addition, panel sessions centered about prominent persons in the field have been organized on: Production Planning and Scheduling, The Future of Operations Research, Effective Presentation of Operations Research Material, Military Air Transportation, and Information Retrieval.

## CATALOGUE OF LECTURE NOTES

### INSTITUTO DE MATEMATICA PURA E APLICADA

The following items published by the Instituto de Matematica Pura e Aplicada, Rua São Clemente 265, Rio de Janeiro, Brazil, are now available:

G. W. MACKEY, Commutative Banach Algebras, 1959, 210 pp.

P. SAMUEL, Elementos de Geometria Algébrica, 1959, 114 pp.

Orders should be addressed to Livraria Castelo, Avenida Erasmo Braga 227, Rio de Janeiro, Brazil.

### INSTITUTO DE FISICA E MATEMATICA

The following items published by the Instituto de Fisica e Matemática, Rua do Hospício 371, Recife, Brazil, which were out of print are now available again:

R. GODEMENT, Variétés Différentiables, 1959, 75 pp.

F. BRUHAT, Algèbres de Lie et Groupes de Lie, 1959, 71 pp.

Orders should be addressed to Livraria Castelo, Avenida Erasmo Braga 227, Rio de Janeiro, Brazil.

## **NEWS ITEMS AND ANNOUNCEMENTS**

BORIS A. BAKHMETEFF RESEARCH FELLOWSHIP IN MECHANICS OF FLUIDS, offered by Humanities Fund, Incorporated, New York, New York, will be available for the 1960-1961 academic year in an amount up to \$3000. It is intended to be a specific contribution for a definite research project of an original and creative nature in the general field of mechanics of fluids.

The recipient shall be a full-time graduate student who is a candidate for the master's or doctoral degree. He shall not hold, or expect to hold, any other fellowship or major income-producing commitment that will interfere with his research work and study on a full-time basis. It is expected that the stipend will be adequate to cover tuition, subsistence, and, if necessary, a portion of the research expenses.

The study and research may be undertaken at an institution of the Fellow's choice. In the judgment of the Committee, the adequacy of the facilities of the institution will have substantial weight in the selection of the Fellow.

Applications should be filed by February 15, 1960 with: Dean William Allan, School of Technology, The City College of New York, New York 31, New York.

NATIONAL SCIENCE FOUNDATION GRADUATE AND POST-DOCTORAL FELLOWSHIPS. The National Academy of Sciences-National Research Council has again been called upon to advise the National Science Foundation in the selection of candidates for the Foundation's program of graduate and postdoctoral fellowships. The Foundation plans to award approximately 1,100 graduate and 125 post-doctoral fellowships in these two programs during the 1960-1961 academic year.

Committees appointed by the Academy-Research Council will evaluate applications of all candidates; final selection will be made by the Foundation and awards announced on March 15, 1960.

Open only to citizens of the United States and awarded solely on the basis of ability, the fellowships may be applied to advanced study in the mathematical, physical, medical, biological, and engineering sciences, including anthropology, psychology (excluding clinical psychology), and the following social sciences: geography, mathematical economics, econometrics, demography, information and communication theory, experimental and quantitative sociology and the history and philosophy of science. They are open to college seniors, graduate and postdoctoral students, and other individuals with equivalent training and experience.

All applicants for graduate (predoctoral) awards will be required

to take an examination designed to test scientific aptitude and achievement. This examination, administered by the Educational Testing Service, will be given on January 16, 1960, at designated centers throughout the United States and certain foreign countries.

The annual stipends for graduate Fellows are as follows: \$1800 for the first year; \$2000 for the intermediate year; and \$2200 for the terminal year. The annual stipend for postdoctoral Fellows is \$4500. Limited allowances will also be provided to apply toward tuition, laboratory fees, and travel.

Further information and application materials may be obtained from the Fellowship Office, National Academy of Sciences-National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C. The deadline for the receipt of applications for regular post-doctoral fellowships is December 22, 1959, and for graduate fellowships, January 1, 1960.

**NSF SUMMER FELLOWSHIPS FOR SECONDARY SCHOOL TEACHERS OF SCIENCE AND MATHEMATICS.** The National Science Foundation has announced a program of Summer Fellowships for Secondary School Teachers of Science and Mathematics. Several hundred awards will be made for study beginning in the summer of 1960 and continuing for as many as three successive summers.

Fellowships will be awarded to support individually planned programs of study in the mathematical, physical, and biological sciences acceptable by the fellowship institution toward an advanced degree in one of these sciences. Although Fellows will not be required to pursue courses of study leading to an advanced degree in science, they will be required to pursue studies at that level.

This Fellowship Program is in addition to the Foundation's support of Institutes for teachers. In the Institutes programs, teachers are afforded opportunities to study in courses of instruction especially designed for groups of teachers. Institute participants are chosen by the staff of the Institute according to locally established criteria. In the Fellowship Program, Fellows may pursue individually planned graduate-level study programs at institutions of their choice. Fellows are selected by the Foundation, solely on the basis of ability, in a national competition.

An applicant for a Summer Fellowship for Secondary School Teachers must: (1) be a citizen of the United States; (2) be a science or mathematics teacher in a secondary school (and must have had three year's experience in that capacity); and (3) hold a baccalaureate degree or its equivalent. Applicants will be evaluated by panels of scientists chosen especially for this task by the American Association for the Advancement of Science. In arriving at their judgments panelists will weigh the applicants demonstrated and potential ability

as a teacher, his academic record, and his ability to pursue a graduate-level program in science or mathematics.

Stipends will be computed at the rate of \$75 for each week of tenure. Travel and dependency allowances will normally be provided, and the Foundation will pay for tuition and required fees.

Teachers should NOT submit their applications directly to the National Science Foundation. Rather, information and application materials may be obtained by addressing a request to Secondary School Fellowships, American Association for the Advancement of Science, 1515 Massachusetts Avenue, N. W., Washington 5, D.C. Completed applications must be received by the Association by January 15, 1960. Names of Fellows selected will be announced March 15, 1960.

FOUNDATION INVITES PROPOSALS FOR SUMMER CONFERENCES. The National Science Foundation announced that it will continue to support in 1960 an experimental program of some 20 Summer Conferences designed primarily for college teachers of engineering, science, and mathematics. The Summer Conferences are of shorter duration than, but generally similar to, the Summer Institutes which the Foundation has successfully supported in recent years.

Colleges and universities interested in sponsoring such conferences are invited to request further information from the Program Director for Summer Institutes, National Science Foundation, Washington 25, D. C. The deadline for submitting proposals is December 15, 1959.

Although planned and directed by the Nation's institutions of higher education, Summer Conferences are supported by Foundation funds to defray direct operational costs plus expenses of participants. Summer Institutes, 362 of which were supported by the Foundation in 1959, are usually from six to ten weeks in length and have been predominantly for high school teachers. In contrast, Summer Conferences extend over a one-to-three-week period and are primarily for college teachers. In a few cases secondary school teachers may be included for particular purposes.

As with Summer Institutes, Summer Conferences are directed toward strengthening teachers' mastery of engineering, science, or mathematics, and increasing their capacity as teachers. A typical Conference might consider a specialized area of engineering, science, or mathematics, or it might be devoted to recent advances in a particular discipline. The material might be presented through short courses or through a series of lectures by visiting specialists. In some cases Conferences might encourage participants to choose specific areas for discussion. Selection of participants is made by the host institutions. College and university staff members wishing to

participate will submit applications directly to the sponsoring school, NOT the Foundation. Announcement of the sponsoring institutions will be made by the National Science Foundation about February 1, 1960.

Conferences and institutes are representative of the several Foundation programs directed toward improving the teaching of engineering, science, and mathematics. Information concerning other Foundation programs for improving science education in the United States can be obtained from the National Science Foundation, Washington 25, D. C.

THE WOODROW WILSON NATIONAL FELLOWSHIP FOUNDATION has announced its 1960-1961 competition for fellowships to be held during the first year of graduate work leading to a career in college teaching. Candidates must be nominated by a college faculty member. Subsequently, through a careful process of selection, including a personal interview, a thousand fellows will be elected from the nominees. Each successful candidate will receive \$1500 plus dependency allowances for wife and children. The Foundation also pays directly to the institution tuition and fees for the fellow.

Any member of the academic profession in any college or university may nominate candidates by sending in the name, college, and address of the candidate to the chairman of the region in which the candidate is currently enrolled as a student. Nominations should be sent as soon as possible but in no case later than October 31, 1959. Names and addresses of regional chairmen are listed in a printed memorandum, copies of which may be secured from the National Director, Woodrow Wilson National Fellowship Foundation, Box 642, Princeton, New Jersey.

RESEARCH INSTRUCTORSHIPS AT WISCONSIN. The University of Wisconsin invites applications for research instructorships or research assistant professorships from young mathematicians who show definite promise in research and teaching. The intent of the award is to allow the recipients to have substantial time for research. The teaching load is just half that of an ordinary staff member on a two-semester average. The number of awards depends on the number of highly qualified applicants. Salary is dependent on qualifications. Appointments are renewable. Application forms may be obtained from the Department of Mathematics.

TEMPORARY MEMBERSHIPS IN THE INSTITUTE OF MATHEMATICAL SCIENCES, NYU. The Institute of Mathematical Sciences at New York University offers temporary memberships to mathema-

ticians and other scientists holding the Ph. D. degree who intend to study and do research in the fields in which the Institute is particularly active. These fields include functional analysis, function theory, differential equations, mathematical physics, fluid dynamics and magnetohydrodynamics, electromagnetic theory and numerical analysis and digital computing.

The temporary membership program is designed primarily to alleviate the present critical shortage of scientists trained in mathematical physics, applied mathematics, and mathematical analysis. The program is being supported by the National Science Foundation and also by funds contributed by industrial firms to New York University.

Temporary members may participate freely in the research projects, the advanced graduate courses, and the research seminars of the Institute, and they will have the opportunity of using the computational facilities.

The temporary members will receive a grant commensurate with their status.

Temporary memberships are awarded for one year, but may be renewed in special cases. Appropriate arrangements can be made for applicants who expect to be on leave of absence from their institutions.

Requests for information and for application blanks should be addressed to the Membership Committee, Institute of Mathematical Sciences, 25 Waverly Place, New York 3, New York.

FELLOWSHIPS AND GRANTS in the humanites, social sciences, and creative arts are catalogued in the September 1959 issue of the ACLS Newsletter, published by the American Council of Learned Societies, 345 East 46th Street, New York City.

THE 1960 HEAT TRANSFER AND FLUID MECHANICS INSTITUTE will be held June 15, 16, 17 at Stanford University. The 1960 Institute continues with the purpose, established during a twelve-year history, of presenting technical and scientific advances in fluid mechanics, heat transfer, and related fields. This year papers are solicited particularly on the following topics: thermal-radiation problems in space technology, fluid problems peculiar to low-gravity environments, rotating fluid systems, flows not in thermodynamic equilibrium, time-dependent viscous flows, separated flows, origins of turbulence, turbulent mixing and diffusion.

Titles and abstracts of proposed papers must be submitted to the chairmen of the Papers Committee, Professor Walter G. Vincenti, or Professor William C. Reynolds, Stanford University, Stanford,

California, by December 1, 1959, and final papers by February 15, 1960.

SECOND INTERNATIONAL SYMPOSIUM ON RAREFIED GAS DYNAMICS. The symposium will range from topics of immediate significance in upper atmosphere and space flight to broad and basic scientific studies. It will be similar in scope and purpose to the 1<sup>er</sup> Symposium International sur l'Aerodynamique et l'Aerothermique des Gaz Rarifiés, held in Nice, France, in July 1958, and will focus attention on new developments since that meeting.

The program of the symposium will include: continuum fluid mechanics and heat transfer at low Reynolds numbers in compressible fluids; studies of the limits of the continuum theory or the quasi-equilibrium kinetic theory of gases; problems in the kinetic theory of gases, particularly attempts to solve the Boltzmann equation; free-molecule and near-free-molecule flow in neutral and ionized gases; the physics of surface interactions between gases and solids; boundary conditions for rarefied gas equations - slip flow; experimental techniques and instrumentation developments bearing on the above, whether applied to laboratory or field experiments.

Attendance at the Symposium will be by invitation only. The second announcement, to be mailed in several months, will contain an application form. If you wish to receive this announcement, please write: Engineering and Sciences Extension, University of California, 2451 Bancroft Way, Berkeley 4, California.

POSTDOCTORAL FELLOWSHIP AWARDS ANNOUNCED BY NATIONAL SCIENCE FOUNDATION. On October 16, 1959, the NSF announced the award of 35 fellowships for advanced study and research in the sciences in the Foundation's Postdoctoral Fellowship Program. All awards were approved by the National Science Board upon the recommendation of Dr. Alan T. Waterman, Director of the Foundation. Only one of the awards was in mathematics. This was to Dr. Paul J. Koosis, formerly affiliated with New York University. The fellowship institution named in the award was Institute Henri Poincare.

SIAM VISITING LECTURESHIPS. The Society for Industrial and Applied Mathematics announces the establishment of a Visiting Scientist Lectureship Program which will serve the dual purpose of familiarizing college and university groups with contemporary mathematical activity in applied and industrial settings, and of making available representative research mathematicians, active in areas of current interest, to industrial groups for visiting lectures and dis-

cussions.

A grant of the National Science Foundation supports the part of the program which is aimed at colleges and universities, making it possible to provide the broadest geographical coverage. The visiting scientists will be prepared to give general lectures, to discuss more specific topics with individual classes and seminars, to advise in individual or group conferences on questions of curriculum, research in progress, or opportunities and requirements in the field of applied mathematics, and generally to cooperate in every way with the colleges in furthering the aims of the program. A normal visit will last for two or three days. Contributions to the program will be sought from the institutions visited in the form of a portion of the traveling expenses and subsistence for the visitor while on campus, but no institution will be denied visits solely because it cannot afford to make such a contribution.

The complementary part of the program will provide opportunities for professional mathematicians who have gone into industry to maintain their ties with current phases of research mathematics. Formal lectures of a survey nature, seminar talks, and non-consultative discussions of personal research interests will constitute the visitor's activity during his normally one or two day visit. It is expected that the inviting organization will in each instance bear the lecturer's costs in full.

The program is under the direction of Dr. F. J. Weyl of the Office of Naval Research, Washington 25, D. C., who is assisted by a committee of the Society appointed for this purpose. Information on the panels of lecturers and on application procedures will be available in the near future.

THE 1959 SUMMER INSTITUTE of the Society, held at the University of Colorado in Boulder during the period June 21-July 18, was devoted to Number Theory. It was supported by the National Science Foundation; some visitors from abroad secured transportation with the Military Air Transport Service.

The following invited addresses were given:

R. C. Bose, "Disproof of Euler's conjecture on Latin squares"

S. Chowla, "Some problems of elementary number theory"

H. Davenport, "Diophantine equations in many variables"

P. Erdős, "Problems and results of elementary additive number theory"

H. B. Mann, "Sums of sets of integers"

H. Rademacher, "Concerning the Riemann-Mangoldt theorem"

A. Selberg, "Some problems concerning discontinuous groups of isometries in higher dimensional symmetric spaces"

In addition, participants gave approximately forty one-hour talks on their current research.

Mathematicians coming from outside the United States included:

B. J. Birch, J. H. H. Chalk, H. Davenport, P. Erdős, R. K. Guy, M. Kneser, L. Moser, G. L. Watson.

Other participants were:

N. C. Ankeny, R. G. Ayoub, P. T. Bateman, A. T. Brauer, W. E. Briggs, R. Caldwell, J. M. Calloway, L. Carlitz, F. M. Carpenter, S. Chowla, P. J. Cohen, M. P. Dolciani, A. L. Duquette, B. M. Dwork, N. J. Fine, P. X. Gallagher, E. Grosswald, J. E. Householder, B. Hunt, K. Iwasawa, S. M. Johnson, J. H. Jordan, R. R. Kelisky, J. B. Kelly, A. Kempner, E. E. Kohlbecker, J. Lehner, W. J. LeVeque, D. J. Lewis, G. G. Lorentz, H. B. Mann, D. C. B. March, P. J. McCarthy, L. R. McCulloh, W. E. Mientka, W. H. Mills, M. Newman, C. A. Nicol, I. Niven, H. Onishi, G. Pall, S. Parameswaran, B. Pollak, E. T. Parker, H. Rademacher, J. Raleigh, J. B. Roberts, J. Robinson, R. M. Robinson, J. B. Rosser, L. A. Rubel, I. Satake, L. Schoenfeld, A. Selberg, J. L. Selfridge, J. E. Shockley, A. Sklar, R. D. Stalley, E. G. Straus, D. Swift, O. T. Todd, J. P. Tull, H. S. Vandiver, R. Z. Vause, M. Ward, A. L. Whiteman, A. C. Woods.

The participants in the Institute resolved that:

The members of the Theory of Numbers Conference wish to make it generally known that they have found the Conference well conceived, well organized and well managed, that they have found it pleasant and enjoyable, and most particularly that they have found it highly profitable as a scientific and mathematical experience. In recognition of these circumstances, they wish to express their appreciation and gratitude to the various groups which participated in the arrangements, specifically the National Science Foundation, which furnished a grant in aid, the American Mathematical Society, which sponsored the Conference, the University of Colorado, which has been most cooperative in supplying facilities, and especially to the Organizing Committee, whose members have given generously of their time and energy both before and during the Conference and whose efforts have been crucial in assuring the success of the Conference.

Moved that:

This resolution be adopted as expressing the sense of the meeting and that the resolution be conveyed to Professor Burton W. Jones with the request that he send copies to members of the Organizing Committee and officials of the other groups named.

RUSSIAN TRANSLATIONS. Two additional volumes in the Society's Russian Translation Series are now available:

Volume 11, Four Papers on Topology:

- Pontryagin, L. S. Smooth manifolds and their applications in homotopy theory. *Trudy Mat. Inst. im Steklov* no. 45, Izdat. Akad. Nauk SSSR, Moscow, 1955.
- Postnikov, M. M. Investigations in homotopy theory of continuous mappings. III. General theorems of extension and classification. *Mat. Sb. N. S.* 40(82) (1956), 415-452.
- Wu, Wen-sün. On Pontryagin classes. III. *Acta. Math. Sinica* 4, (1954), 323-346. (Chinese)
- Bokštejn, M. F. Homology invariants of topological spaces. *Trudy Moskov. Obšč.* 5 (1956), 3-80; *ibid.* 6 (1957), 4-133.

385 pages, list price \$5.90, price to AMS members \$4.43.

Volume 12, Twelve Papers on Analysis, Probability, and Topology:

- Kreĭn, M. G. The ideas of P. L. Čebyšev and A. A. Markov in the theory of limiting values of integrals and their further development. *Uspehi Mat. Nauk (N.S.)* 6 (1951), no. 4(44), 3-120.
- Kreĭn, M. G. and Rehtman, P. G. Development in a new direction of the Čebyšev-Markov theory of limiting values of integrals. *Uspehi Mat. Nauk (N.S.)* 10 (1955), no. 1 (63), 67-78.
- Kudryavcev, I. D. On implicit functions. *Uspehi Mat. Nauk (N.S.)* 9 (1954), no. 3 (61), 155-156.
- Stesin, I. M. Computation of eigenvalues by means of continued fractions. *Uspehi Mat. Nauk (N.S.)* 9 (1954), no. 2 (60), 191-198.
- Stesin, I. M. An estimate of the precision of computation of eigenvalues by means of continued fractions. *Vičisl. Mat. Vičisl. Tehn.* 2 (1955), 145-150.
- Krasnosel'skiĭ, M. A. On some methods of approximate calculation of the characteristic values and characteristic vectors of a positive definite matrix. *Uspehi Mat. Nauk (N.S.)* 11 (1956) no. 3(69), 151-158.
- Gel'fond, A. O. On estimation of certain determinants and the application of these estimations to the distribution of eigenvalues. *Mat. Sb. N. S.* 39(81) (1956), 3-22.
- Hincin, A. Ya. The concept of entropy in the theory of probability. *Uspehi Mat. Nauk (N.S.)* 8 (1953), no. 3(55), 3-20.

- Gel'fand, I. M. and Yaglom, A. M. Calculation of the amount of information about a random function contained in another such function. *Uspehi Mat. Nauk (N.S.)* 12 (1957) no. 1(73), 3-52.
- Chiang, Tse-Pei. Remark on the definition of the quantity of information. *Teor. Veroyatnost. i Primenen* 3 (1958), 99-103.
- Dynkin, E. B. Homologies of compact Lie groups. *Uspehi Mat. Nauk (N.S.)* 8 (1953), no. 5(57), 73-120; 9 (1954), no. 2(60), 233.
- Dynkin, E. B. Topological characteristics of homomorphisms of compact Lie groups. *Mat. Sb. N. S.* 35(77) (1954), 129-173.

342 pages, list price \$4.60, price to AMS members \$3.45.

A catalogue of the entire series is available upon request to the Headquarters Offices of the Society.

HANDBOOK FOR AUTOMATIC COMPUTATION. Preparation of a handbook for automatic computation, in five or more volumes, is now under way for publication by Springer-Verlag. It will appear in F. K. Schmidt's series, "Grundlehren der Mathematischen Wissenschaften." Editors are:

F. L. Bauer, Mainz	H. Rutishauser, Zurich
A. S. Householder, Oak Ridge	K. Samelson, Mainz
F. W. J. Olver, Teddington	R. Sauer, Munich
E. Stiefel, Zurich	

The purpose of the handbook is to provide a collection of tested algorithms for mathematical computations of all sorts: the solution of finite and of functional equations, methods of approximating functions, the evaluation of special functions, etc. These algorithms are to be written in Algol, hence will be usable on any machine for which a suitable translator is available, and even without a translator can be used as a model for programming. It is evident that such a collection could have no general utility unless written in some common program language. The descriptive language will be English.

As plans now stand, the organization of the series will be as follows: Volume 1a will contain a description of the use of Algol, and Volume 1b a description of the structure of translators. These introductory volumes are the only ones that will not be made up primarily of actual algorithms. Volume 2 will be devoted to the solution of finite equations, linear and nonlinear, including the determination of characteristic values and vectors of matrices. Volume 3 will be on functional equations, especially differential equations, ordinary and partial, and integral equations. Volume 4 is concerned with methods

of approximation, and Volume 5 the evaluation of particular functions. It is possible that certain algorithms, such as those for solving inequalities, for mathematical programming, for statistical computations, and the like, that do not seem to fall naturally in any of these areas, may be reserved for a sixth volume. Each algorithm is to be accompanied by enough explanatory information to make it understandable, along with whatever information is available on speed, accuracy, range, or, more generally, for judging the effectiveness of the algorithm for a given type of problem. In any event, only pretested algorithms will be published.

Before the appearance of the volumes themselves, the algorithms will be prepublished in a series of supplements to the journal, "Numerische Mathematik." This is partly to make generally available each algorithm at the earliest possible time. But in addition to this, it provides the possibility for including in the handbook itself additional information, and even corrections, that might come in from users.

Contributions are earnestly solicited. For the present, at least, these must necessarily be in the form of actual algorithms, along with information as to the extent and mode of testing the algorithm, estimates of accuracy, and experience in using it. Untested algorithms will not necessarily be rejected ipso facto, but their use must necessarily await actual test. As algorithms are published, information relating to published algorithms also will be welcomed. Contributions may be sent to any of the editors named above.

THE ANNUAL LIST OF CHAIRMEN OF DEPARTMENTS has been compiled. Copies may be obtained free of charge by writing the Headquarters Offices, 190 Hope Street, Providence 6, Rhode Island.

DR. SOLOMON LEFSCHETZ, received an honorary degree from the University of Paris on Friday, October 16. He was quoted in press releases on this occasion as saying that Soviet successes in rocket guidance can be traced to their lead in certain basic mathematics. Now 75, Moscow-born Dr. Lefschetz became one of America's leading mathematicians despite accidental loss of both hands and forearms in his youth. He left retirement last year to organize a staff of specialists at RIAS, a division of The Martin Company near Baltimore, in mathematical studies that underlie understanding of rocket control systems.

RUTGERS UNIVERSITY STATISTICS CENTER, which will be the State University's central unit for research and teaching in the

field of statistics, was recently established. It will have responsibility for a program of study leading to the Master's Degree or the Ph. D. in Applied and Mathematical Statistics.

The Center, which is under the administration of Dr. Marion A. Johnson, dean of the Graduate School, will be directed by Dr. Ellis R. Ott, who has previously served as professor of Mathematics and Chairman of the Mathematics Department of University College, as well as Chairman of the Rutgers program in Applied and Mathematical Statistics. Director of Research at the Center will be Dr. Martin B. Wilk. The staff will also include Dr. Roger S. Pinkham, Dr. Mason E. Wescott and Harold F. Dodge.

Since 1952, Rutgers has had course work leading to a Master's degree in the field of Applied and Mathematical Statistics. Through June, 1959, a total of 85 Master's degrees in applied and mathematical statistics had been awarded, mostly to full-time employees in nearby industries. The present development constitutes a major extension of the Rutgers Statistics Program plus an administrative re-organization.

UCLA has scheduled, at the start of the current fall semester, over 30 courses in computer techniques and uses, including courses in data processing, analog and digital systems, circuit design, and programming.

## 190 HOPE STREET

September 22, 1959 was a memorable day in the history of the Society, for then the Society became the owners of the property, including the Headquarters Office Building, at 190 Hope Street, Providence. This was the culmination of extensive planning and many negotiations begun in the fall of 1955, almost four years before.

At that time the Society's offices were located in a building at 80 Waterman Street, rented from Brown University. The Headquarters' staff of the rapidly growing Society had outgrown the available space in that location.

A search was begun for new quarters. It soon developed that the Chace mansion at 190 Hope Street (the southeast corner of Hope and Waterman) was potentially available. Albert E. Meder, Jr., the Treasurer, and John H. Curtiss, then Executive Director of the Society, visited the premises and initiated the plans to acquire the property.

There were, however, complications to be overcome. Under the terms of the Chace estate, Brown University had a first option to purchase the property. Furthermore, it developed that though the Society is tax-exempt under provisions of the Federal tax laws, it would require a special act of the Rhode Island legislature before the Society could own tax exempt property in Rhode Island.

As on many occasions in the past, Brown University again lent a helping hand. The University arranged to purchase the property and again to serve as landlord to the Society until satisfactory arrangements could be made for the Society to acquire the property.

The required legislation was introduced at the 1958 session of the General Assembly of Rhode Island, was passed, but was vetoed by the then governor. It was thus necessary to delay arrangements for another year. Then, through the astute management of John Curtiss, the legislation on the Society's behalf was passed and signed by the governor. At last it was possible to proceed to the final arrangements so that the Society could purchase and acquire title to the property.

The Society's home is conveniently located in Providence. Within a radius of a few blocks are found Brown's Mathematics Department, the History of Mathematics Department, the Division of Applied Mathematics, the Library (which is so important to the MATHEMATICAL REVIEWS staff), and the Brown University Computing Center now under construction.

Members of the Society are frequent visitors at 190 Hope Street and, needless to say, the Headquarters Staff appreciate these visits and the opportunity they afford to further acquaint the members with the Society's activities.

190 HOPE STREET





## PERSONAL ITEMS

(This section is restricted to members of the Society)

Professor S. P. Diliberto and Associate Professor R. J. DeVogelaere of the University of California, Berkeley, have been invited and will address the Symposium on Ordinary Differential Equations, September 1959, at the National University of Mexico in Mexico City.

Professor L. A. Henkin, Professor Alfred Tarski, and Assistant Professor R. L. Vaught of the University of California, Berkeley, have been invited to attend and address the International Symposium on Foundations of Mathematics in Warsaw, Poland in September 1959.

Associate Professor E. A. Bishop of the University of California, Berkeley, will continue on his Sloan Foundation Fellowship which will relieve him of half of his teaching duties.

Associate Professor J. W. Brace, on leave from the University of Maryland, has received a National Science Foundation Fellowship and will spend the year at the University of California, Berkeley.

Associate Professor C. M. Braden, on leave from Macalester College, has received a National Science Foundation Science Faculty Fellowship and will spend the 1959-1960 academic year at the University of California, Berkeley.

Professor J. A. Clarkson of Tufts University has been awarded a National Science Foundation Science Faculty Fellowship and will spend the fall term at the University of California, Berkeley.

Associate Professor H. O. Cordes of the University of California, Berkeley, has been awarded a Sloan Foundation Fellowship which will relieve him of half of his teaching duties for two years, 1959-1961.

Mr. L. E. Dubins of the Institute for Advanced Study has been awarded a National Science Foundation Fellowship and will spend the 1959-1960 academic year at the University of California, Berkeley.

Assistant Professor C. C. Faith, on leave from Pennsylvania State University, has been awarded a NATO Fellowship and is spending the 1959-1960 academic year at the Mathematisches Institut, Heidelberg, Germany.

Dr. G. H. Golub of the University of Illinois is at the University of Cambridge, England, as the recipient of a National Science Foundation Fellowship award.

Professor Emeritus Solomon Lefschetz of Princeton University has been awarded the title of Doctor honoris causa by the Council of the University of Paris.

Professor Janet McDonald, on leave from Vassar College, has received a National Science Foundation Faculty Fellowship award and will be at Indiana University for the 1959-1960 academic year.

Assistant Professor J. W. Milnor, on leave from Princeton Uni-

versity, has been appointed to a visiting professorship at the University of California, Berkeley. He will be half time on a Sloan Foundation Fellowship at the University of California.

Dr. R. C. Sacksteder of Johns Hopkins University has received a National Science Foundation Post Doctoral Fellowship and will be at Yale University for the 1959-1960 academic year.

Assistant Professor J. W. Addison, Jr. of the University of Michigan has been appointed Lecturer at the University of California, Berkeley.

Mr. W. F. Ames of E. I. du Pont de Nemours and Company has been appointed to a research assistant professorship at the University of Delaware.

Dr. D. R. Anderson of RAND Corporation has accepted a position as Member of the Technical Staff of Hughes Aircraft Company, Culver City, California.

Dr. K. I. Appel of the University of Michigan has accepted a position as Mathematician with the Institute for Defense Analyses, Princeton, New Jersey.

Associate Professor J. H. Barrett, on leave from the University of Utah, has been appointed to an associate professorship in the U. S. Army Mathematics Research Center, University of Wisconsin, Madison, Wisconsin.

Dr. Lida K. Barrett, on leave from the University of Utah, has been appointed Lecturer at the University of Wisconsin.

Professor Grace E. Bates, on leave from Mount Holyoke College, has been appointed to a visiting professorship at Dartmouth College.

Dr. N. W. Bazley of the National Bureau of Standards has been appointed to an assistant research professorship at the University of Maryland.

Assistant Professor H. S. Bear, on leave from the University of Washington, has been appointed to a visiting assistant professorship at Princeton University.

Associate Professor Gerald Berman of Illinois Institute of Technology has been appointed to a professorship at the University of Waterloo.

Dr. R. L. Bishop of Massachusetts Institute of Technology has been appointed to an assistant professorship at the University of Illinois.

Associate Professor J. H. Blau, on leave from Antioch College, will be at Stanford University for the 1959-1960 academic year.

Dr. I. E. Block of Burroughs Corporation has accepted the position of Manager, UNIVAC Engineering Computer Center, Remington-Rand Division, Sperry Rand Corporation, Philadelphia, Pennsylvania.

Assistant Professor J. R. Blum of Indiana University has ac-

cepted a position as Member of the Technical Staff, Sandia Corporation, Albuquerque, New Mexico.

Professor R. C. Bose, on leave from the University of North Carolina, has been appointed to a visiting university professorship at Case Institute of Technology.

Dr. J. H. Bramble of General Electric Company has accepted a position as Mathematician with the United States Naval Ordnance Laboratory, Silver Spring, Maryland.

Dr. T. F. Bridgland, Jr. of the University of Florida has accepted a position as Research Specialist with Boeing Airplane Company, Seattle, Washington.

Mr. D. M. Burton of the University of Rochester has been appointed to an assistant professorship at the University of New Hampshire.

Professor S. S. Cairns, on leave from the University of Illinois, will be at the Institute for Advanced Study for the 1959-1960 academic year.

Mr. T. W. Cairns of Oklahoma State University has been appointed to an assistant professorship at the University of Tulsa.

Mr. W. V. Caldwell of the University of Michigan has been appointed to an assistant professorship at the University of Delaware.

Professor Y. W. Chen, on leave from Wayne State University, has been appointed a Temporary Member of the Institute of Mathematical Sciences, New York University.

Dr. P. J. Cohen of Massachusetts Institute of Technology has been appointed a Member of the Institute for Advanced Study.

Associate Professor Geoffrey Crofts of Occidental College has accepted a position as Actuarial Training Director with Occidental Life Insurance Company of California, Los Angeles, California.

Associate Professor M. D. Davis of Rensselaer Polytechnic Institute has been appointed to a visiting associate professorship at New York University.

Mr. E. R. Deal of the University of Michigan has been appointed to an assistant professorship at Colorado State University.

Professor Johannes de Groot, on leave from the University of Amsterdam, has been appointed to a visiting professorship at Purdue University.

Professor René Deheuvels, on leave from the University of Lille, has been appointed to a visiting professorship at Yale University.

Mr. H. H. Diehl of Southern Union College has been appointed to an assistant professorship at Wittenberg University.

Assistant Professor R. L. Disney of Lamar State College of Technology has been appointed to an associate professorship at the University of Buffalo.

Dr. R. E. Dowds of Purdue University has been appointed to an associate professorship at Butler University.

Dean W. H. Durfee of Hobart and William Smith Colleges has retired with the title Professor Emeritus.

Dr. Edmund Eisenberg of Brown University has accepted a position as Member of the Senior Staff of Hughes Aircraft Company, Culver City, California.

Dr. George Epstein of Hughes Aircraft Company has accepted a position as Member of the Technical Staff of Gilfillan Brothers, Los Angeles, California.

Dean Frances E. Falvey of Millikin University has been appointed Dean at Milwaukee-Downer College.

Mr. M. H. Farrant of Coates, Herfurth and England has accepted a position as Chief Actuary with British Pacific Life Insurance Company, Vancouver, British Columbia, Canada.

Professor F. A. Ficken of the University of Tennessee has been appointed to a professorship at New York University.

Assistant Professor J. E. Forbes, on leave from Purdue University, has been appointed to a visiting professorship at Ball State Teachers College.

Professor Emeritus Tomlinson Fort of the University of Georgia has been appointed to a visiting professorship at Emory University.

Assistant Professor D. J. Foulis of Lehigh University has been appointed to an assistant professorship at Wayne State University.

Associate Professor K. D. Fryer of the Royal Military College of Canada has been appointed to a professorship at the University of Waterloo.

Professor P. R. Garabedian of Stanford University has been appointed to a professorship at New York University.

Dr. M. A. Geraghty of the University of Notre Dame has been appointed an Office of Naval Research Post-Doctoral Research Associate at Northwestern University.

Mr. E. H. Greene of the University of Virginia has been appointed to an assistant professorship at Southwestern at Memphis.

Professor Emeritus F. L. Griffin of Reed College has been appointed to a visiting professorship at Wesleyan University.

Professor S. W. Hahn of Winthrop College has been appointed to a professorship at Hampden-Sydney College.

Assistant Professor F. C. Hall of Manhattan College has been appointed to an assistant professorship at St. John's University, New York.

Associate Professor Haim Hanani, on leave from the Israel Institute of Technology, has been appointed to an associate professorship at the Mathematics Research Center of the University of Wisconsin, Madison.

Assistant Professor Jun-ichi Hano, on leave from Nagoya University, has been appointed to an assistant professorship at the University of Washington.

Reverend F. A. Homann of Loyola College has been appointed to a research assistantship at Woodstock College.

Associate Professor L. Aileen Hostinsky of Pennsylvania State University was a visiting professor at Syracuse University during the 1959 summer session.

Mr. J. L. Hursch of Bendix Aviation Corporation has been appointed to an assistant professorship at San Diego State College.

Professor H. D. Huskey, on leave from the University of California, Berkeley, will spend the 1959-1960 academic year in Europe.

Mr. E. H. Jacobs of RAND Corporation has accepted a position as Head of the Data Processing Research Staff of System Development Corporation, Santa Monica, California.

Professor J. A. Jenkins of the University of Notre Dame has been appointed to a professorship at Washington University.

Professor P. B. Johnson of Occidental College has been appointed to an associate professorship at the University of California, Los Angeles.

Professor B. W. Jones, on leave from the University of Colorado, has been appointed to a visiting professorship at the University of Puerto Rico, Mayaguez, Puerto Rico.

Dr. W. J. Kammerer of the University of Wisconsin has been appointed to an assistant professorship at Case Institute of Technology.

Miss Dora E. Kearney of the University of Maryland has been appointed to an assistant professorship at Mississippi State University.

Dr. J. E. Keisler of the University of Michigan has been appointed to an assistant professorship at Louisiana State University.

Assistant Professor C. E. Kerr of Lafayette College has been appointed to an assistant professorship at Dickinson College.

Dr. H. S. Kieval of the Polytechnic Institute of Brooklyn has been appointed to an associate professorship at the State University of New York College of Education at New Paltz.

Mr. D. B. Kirk of Curtiss-Wright has been appointed an Associate Research Mathematician at the Willow Run Laboratories of the University of Michigan.

Assistant Professor J. E. Kist of Wayne State University has been appointed to an assistant professorship at Pennsylvania State University.

Professor Klaus Krickeberg, on leave from the University of Heidelberg, has been appointed to a visiting professorship at the University of Aarhus, Denmark.

Professor Hans Lewy, on leave from the University of California, will be at New York University for the 1959-1960 academic year.

Associate Professor A. J. Lohwater of the University of Michigan has been appointed to an associate professorship at Rice Institute.

Dr. R. D. Luce of Harvard University has been appointed to a professorship at the University of Pennsylvania.

Professor R. C. Lyndon, on leave from the University of Michigan, will be at the Institute for Defense Analyses, Princeton, New Jersey for the 1959-1960 academic year.

Professor Emeritus R. B. McClennon of Grinnell College has been appointed to a visiting professorship at the University of Corpus Christi.

Mr. T. H. MacGregor of Rutgers, The State University, has been appointed to an assistant professorship at Lafayette College.

Dr. J. E. Mack of Purdue University has been appointed to an assistant professorship at Ohio University.

Dr. W. S. Mahavier of Illinois Institute of Technology has been appointed to an assistant professorship at the University of Tennessee.

Assistant Professor Karl Matthies of the University of Cincinnati has been appointed to an associate professorship at the University of South Carolina.

Dr. R. D. Mayer of the University of Washington has been appointed to an assistant professorship at Idaho State College.

Assistant Professor E. P. Merkes of DePaul University has been appointed to an assistant professorship at Marquette University.

Dr. R. J. Mihalek of Illinois Institute of Technology has been appointed to an assistant professorship at the University of Wisconsin, Milwaukee.

Dr. R. A. Moore of Lackland Air Force Base has been appointed to an assistant professorship at St. Mary's University, San Antonio, Texas.

Professor Emeritus L. J. Mordell of Cambridge University has been appointed to a visiting professorship at the University of Colorado.

Assistant Professor Anne C. Morel, on leave from the University of California, Davis, will be a Member of the Institute for Advanced Study for the 1959-1960 academic year.

Professor C. B. Morrey, Jr. is on leave from the University of California, Berkeley, to accept an appointment with the Miller Institute for Basic Research.

Miss Mildred Nelson of the Technical Research Group has accepted a position as Senior Mathematician with System Development Corporation, Lodi, New Jersey.

Dr. J. W. Neuberger of Illinois Institute of Technology has been appointed to an assistant professorship at the University of Tennessee.

Dr. E. S. O'Keefe of New York University has accepted a position as Research Specialist with Boeing Airplane Company, Seattle, Washington.

Mr. E. M. Olson of Aberdeen Proving Ground has accepted a position as Mathematician with Radio Corporation of America, Moorestown, New Jersey.

Dr. C. C. Oursler of Indiana University has been appointed to an assistant professorship at the Southern Illinois University Residence Center, East St. Louis, Illinois.

Assistant Professor R. E. Ozimkoski of Fordham University has been appointed to an assistant professorship at Merrimack College.

Dr. Hiram Paley of the University of Wisconsin has been appointed to an assistant professorship at the University of Illinois.

Associate Professor W. W. Peterson, on leave from the University of Florida, has been appointed to a visiting professorship at Massachusetts Institute of Technology.

Dr. R. S. Pinkham of Princeton University has been appointed to an associate professorship at Rutgers, The State University.

Assistant Professor E. E. Posey of West Virginia University has been appointed to an associate professorship at Virginia Polytechnic Institute.

Professor G. B. Price, on leave from the University of Kansas, has been appointed to a visiting professorship at California Institute of Technology.

Dr. A. L. Rabenstein, Jr. of the University of Wisconsin has been appointed to an assistant professorship at Allegheny College.

Dr. Rimhak Ree of the University of British Columbia has been appointed to a research associateship at Columbia University.

Dr. P. B. Richards of Thompson Ramo-Wooldridge Incorporated has accepted a position as Space Mechanics Physicist with General Electric Company, Philadelphia, Pennsylvania.

Dr. R. A. Roberts of Westinghouse Electric Corporation has been appointed to an associate professorship at Ohio Wesleyan University.

Mr. D. A. Robinson of the University of Wisconsin has been appointed to an assistant professorship at Georgia Institute of Technology.

Dr. J. C. Rogers of Johns Hopkins University has accepted a position as Operations Analyst with the Stanford Research Institute, Menlo Park, California.

Professor Helmut Röhrl, on leave from the University of Munich, has been appointed to a visiting associate professorship at the University of Minnesota.

Dr. J. J. Rothman of the University of Chicago has been appointed a research associate at the University of Illinois.

Dr. B. H. Sams of Dartmouth College has accepted a position as Mathematician with Radio Corporation of America, Princeton, New Jersey.

Professor H. E. Scarf, on leave from Stanford University, has been appointed to the Staff of the Cowles Foundation for Research in Economics at Yale University.

Professor Peter Scherk of the University of Saskatchewan has been appointed to a professorship at the University of Toronto.

Dr. Hans Schneider of Queen's University has been appointed to an assistant professorship at the University of Wisconsin, Madison.

Associate Professor B. M. Seelbinder of the University of Alabama has been appointed to an associate professorship at Wake Forest College.

Dr. R. G. Selfridge of the Naval Ordnance Test Station has been appointed an Associate Professor and Director of the Computing Facility at Miami University, Oxford, Ohio.

Mr. R. A. Sibley, Jr. of the University of Texas has accepted a position as a programmer with International Business Machines Corporation, Kingston, New York.

Mr. Robert Simon of Fairchild Engine and Airplane Corporation has accepted a position as Staff Analyst with the Remington Rand Division of Sperry Rand Corporation, New York, New York.

Professor Vikramaditya Singh, on leave from D. A. V. College, has been appointed a Research Associate at Harvard University.

Dr. C. V. L. Smith of the United States Army has been appointed Chief of the Data Systems Division, Goddard Space Flight Center, National Aeronautics and Space Administration, Washington, D. C.

Professor R. F. Smith of the State University of New York College of Education at Oswego has been appointed to a professorship at Earlham College.

Dr. J. J. Sopka, Jr. of International Business Machines Corporation has accepted a position as Mathematician with the National Bureau of Standards, Commerce Department, Boulder, Colorado.

Professor Emeritus R. C. Staley of the University of North Dakota has been appointed to a visiting professorship at Macalester College.

Associate Professor Emeritus Ruth W. Stokes of Syracuse University has been appointed to an associate professorship at Longwood College.

Assistant Professor Ralph Surasky of North Georgia College has been appointed to an assistant professorship at the Extension Center of the University of South Carolina, Florence, South Carolina.

Dr. Roy Takenaga of the University of California has been appointed to an assistant professorship at Sacramento State College.

Mrs. Nancy H. Tapper of Michigan State University has been appointed Lecturer at Cornell University.

Professor Alfred Tarski of the University of California, Berkeley, will be on sabbatical leave, Fall 1959, and will complete his term at the Miller Institute for Basic Research in the Spring 1960.

Dr. O. E. Taulbee of Lockheed Aircraft Corporation has been appointed to an associate professorship in the Oakland Branch of Michigan State University.

Dr. G. T. Thompson of Burroughs Corporation has accepted a position as Mathematical Specialist with Philco Corporation, Palo Alto, California.

Dr. E. W. Titt of CONVAIR has been appointed to a professorship at the University of Arizona.

Assistant Professor J. W. Toole of St. Peter's College has been appointed to an assistant professorship at the University of Maine.

Dr. L. E. Ward, Jr. of the Naval Ordnance Test Station, has been appointed to an associate professorship at the University of Oregon.

Dr. R. J. Warne of the University of Tennessee has been appointed to an assistant professorship at Louisiana State University, New Orleans, Louisiana.

Dr. A. D. Wasel of Lockheed Aircraft Corporation has been appointed to a professorship at the University of San Diego.

Associate Professor Robert Weinstock, on leave from the University of Notre Dame, has been appointed to a visiting associate professorship at Oberlin College.

Dr. L. R. Welch, on leave from California Institute of Technology, has accepted a position as Mathematician with the Institute for Defense Analyses, Princeton, New Jersey.

Assistant Professor Chien Wenjen of Texas Technological College has been appointed to an assistant professorship at Long Beach State College.

Mr. R. J. Wernick of New York University has accepted a position as Mathematician with General Electric Company, Schenectady, New York.

Professor Emeritus L. T. Wilson of the United States Naval Academy has been appointed to a professorship at Jacksonville University.

Professor Emeritus R. M. Winger of the University of Washington has been appointed Lecturer at the University of Arizona.

Professor František Wolf of the University of California will be on sabbatical leave for the Fall term, 1959.

Dr. C. R. B. Wright of the University of Wisconsin will be a Research Fellow at California Institute of Technology.

Dr. P. B. Yale of Harvard University has been appointed to an assistant professorship at Oberlin College.

Mr. A. W. Yonda of AVCO Manufacturing Corporation has accepted a position as Senior Member, Technical Staff, Radio Corporation of America, New York, New York.

Assistant Professor L. N. Zaccaro of the University of Rhode Island has been appointed to an associate professorship at Hiram College.

The following promotions are announced:

B. H. Arnold, Oregon State College, to a professorship.

W. G. Bade, University of California, Berkeley, to an associate professorship.

E. A. Bishop, University of California, Berkeley, to an associate professorship.

V. W. Bolie, Iowa State University, to a professorship.

W. E. Briggs, University of Colorado, to an associate professorship.

F. E. Browder, Yale University, to an associate professorship.

H. O. Cordes, University of California, Berkeley, to an associate professorship.

W. F. Darsow, DePaul University, to an associate professorship.

C. A. Davis, University of California, Davis, to a professorship.

R. A. Dean, California Institute of Technology, to an associate professorship.

S. P. Diliberto, University of California, Berkeley, to a professorship.

F. B. Fuller, California Institute of Technology, to an associate professorship.

L. E. Fuller, Kansas State University of Agriculture and Applied Science, to a professorship.

F. W. Gehring, University of Michigan, to an associate professorship.

L. W. Green, University of Minnesota, to an associate professorship.

J. C. Kiefer, Cornell University, to a professorship.

Bertram Kostant, University of California, Berkeley, to an associate professorship.

R. C. Lyndon, University of Michigan, to a professorship.

Janet McDonald, Vassar College, to a professorship.

N. G. Meyers, University of Minnesota, to an assistant professorship.

H. C. Miller, Jr., University of Alabama, to an assistant professorship.

G. D. Mock, State University of New York College of Education at Oswego, to a professorship.

R. A. Moreland, Jr., Texas Technological College, to an assistant professorship.

Benjamin Muckenhoupt, DePaul University, to an assistant professorship.

M. W. Oliphant, Georgetown University, to an associate professorship.

J. M. Osborn, Jr., University of Wisconsin, to an assistant professorship.

C. H. Papas, California Institute of Technology, to a professorship.

Edmund Pinney, University of California, Berkeley, to a professorship.

J. R. M. Radok, Polytechnic Institute of Brooklyn, to a professorship.

Michio Suzuki, University of Illinois, to a professorship.

Dorothy M. Swan, State University of New York College of Education at Cortland, to an associate professorship.

H. J. Weinitschke, University of California, Los Angeles, to an assistant professorship.

G. L. Weiss, DePaul University, to an associate professorship.

The following appointments to instructorships are announced:

Alabama College: Mr. D. C. Stevens; University of Arkansas: Mrs. Jeanne C. Gardner; Cornell University: Dr. R. H. Farrell; DePaul University: Dr. Richard O'Neil, Dr. R. H. Szczarba, Dr. J. Z. Yao; Duke University: Dr. H. E. Debrunner, Dr. R. T. Harris; Harvard University: Dr. Adam Koranyi; Illinois Institute of Technology: Mr. Sidney Penner; Miami University: Mr. Kermit Hutcher-son; University of Michigan: Dr. G. W. Hedstrom, Dr. J. M. Kister; Ohio Wesleyan University: Mr. G. M. Nielsen; University of Oregon: Mr. F. L. McMains, Jr.; Purdue University: Mr. N. S. Scarritt, Jr.; University of Rochester: Mr. B. L. Lercher; University of Saskatchewan: Dr. P. M. Cuttle, Dr. Yvonne H. Cuttle; Stanford University: Dr. P. C. Fife; Tufts University: Mr. Paul Aizley; Wake Forest College: Mr. E. W. Womble; University of Washington: Dr. W. B. Woolf; Wellesley College: Mr. R. J. Crittenden; University of Wisconsin: Dr. D. S. Ornstein; University of Wisconsin, Milwaukee: Mr. R. A. Kurtz; Worcester Polytechnic Institute: Mr. G. C. Branche; Yale University: Dr. Louis Brickman, Dr. D. W. Dean, Dr. A. C. Mewborn.

#### Deaths:

Professor Emeritus J. A. Bullard of the University of Vermont died on April 10, 1959 at the age of 72 years. He had been a member of the Society for 38 years.

Dr. Charles S. Fazel of Santa Cruz, California died last July at the age of 66 years. He had been a member of the Society for 21 years.

Professor Emeritus R. E. Gaines of the University of Richmond died on June 19, 1959 at the age of 98 years. He had been a member of the Society for 32 years.

# MEMORANDA TO MEMBERS

## THE EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register, established by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, will be maintained at the Winter Meeting in Chicago, Illinois, on January 27-29, 1960. The Register will be conducted from 9:00 A.M. to 5:00 P.M. on January 27, 28, and 29.

The Placement Service Desk will be located on the third floor of the Conrad Hilton Hotel. Interview tables in communal rooms will be assigned for interviews. There will be no charge for registering to either job applicants or to employers, except when the Late Registration Fee for employers is applicable. Provision will be made for anonymity of applicants upon request.

Job applicants and employers wishing to be listed will please write to the Employment Register, 190 Hope Street, Providence 6, Rhode Island, for application forms and for position description forms, which must be completed and returned to Providence not later than January 8, 1960, in order to be included in the listings at the meeting in Chicago free of charge. Forms which arrive after this closing date, but before January 26, will be included in the listings at the meeting for a Late Registration Fee of \$3.00 and will also be included in the sold listings, but not until ten days after the meeting. Printed listings will be available for sale both during and after the meeting. The prices are as follows: Position descriptions, \$2.00; listing of applicants, academic only, \$5.00; comprehensive listing of applicants, academic, industrial, and government, \$20.00.

It is essential that applicants and employers register at the Placement Service Desk promptly upon arrival at the meeting to receive the call number which has been assigned to them.

## SUMMER EMPLOYMENT FOR MATHEMATICIANS and COLLEGE MATHEMATICS STUDENTS

The Providence office of the Society plans to compile a list of opportunities for summer employment for mathematicians and college mathematics students. Institutions and industrial firms which would welcome inquiries for summer employment for the year 1960 from mathematicians and college mathematics Students should please write to the Employment Register, American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, indicating the number of openings, description of position, and officer to whom application should be directed.

## THE AMS-IMS TRANSLATION PROGRAM

Suggestions are invited for mathematical articles in foreign languages, particularly in Russian and Chinese, to be translated and published by the Society. Articles of recent interest are chiefly desired but suggestions for older articles will be welcome even though they may have been submitted before. The Society hopes to continue in 1960-1961 its present program of publishing about 2,000 pages, of which about 500 will be in the fields of Probability and Statistics. At the recent meeting in Salt Lake City the Council recommended the translation of books. Nominations for inclusion in this program are particularly welcome. Suggestions should be sent to

Russian Translation Project  
American Mathematical Society  
190 Hope Street  
Providence 6, Rhode Island.

## RECIPROCITY AGREEMENT with the MATHEMATICAL SOCIETY OF JAPAN

The American Mathematical Society has entered into a reciprocity agreement with the Mathematical Society of Japan, by which members of each may become members of the other by paying half the regular dues. The regular dues of the Mathematical Society of Japan are 1800 Yen, amounting to roughly five dollars. Members receive two periodicals. One of these, the Journal of the Mathematical Society of Japan has four issues per year of about 120 pages of original mathematical papers written in European Languages. The other, the Sagaku, has four 170 page issues of expository papers and news items published by the Society. Those members of the American Mathematical Society wishing to join the Mathematical Society of Japan, or being presently members of the latter Society and wishing to take advantage of the reciprocity agreement with respect to dues, should write

President, Mathematical Society of Japan  
c/o Faculty of Science  
University of Tokyo  
Tokyo, Japan.

It is understood that members under reciprocity agreement spending time in the other country should pay the regular dues while they are there.

## NEW PUBLICATIONS

- Alexandroff, P. Die topologischen Dualitätssätze. I. Abgeschlossene Mengen. (Mathematische Forschungsberichte, VII.) Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 106 pp. Paperbound, 16.80 DM.
- Azad, A.K. Elementary analysis of the turbulent plate layer. Washington, Catholic University of America, 1959. 3 + 10 pp.
- Beckmann, M. J. Lineare Planungsrechnung, Bd. 1. With a preface by E. Gutenberg. (Wirtschaftswissenschaft der Gegenwart, I. Planungsforschung.) Ludwigshafen am Rhein, Fachverlag für Wirtschaftstheorie und Ökonometrie, 1959. 10 + 118 pp. 13.80 DM.; paperbound, 9.80 DM.
- Boscher, J. Résolution par analogie électrique d'équations aux dérivées partielles du quatrième ordre intervenant dans divers problèmes d'élasticité. Preface by J. Pérès. (Publ. Sci. Tech. Ministère d l'Air, no. 348.) Paris, 1958. 21 + 130 pp. 2251 fr.
- Brenner, J. L. See Gantmacher, F. R.
- Bushaw, D. W. See Gantmacher, F. R.
- Carnap, R. See Reichenbach, H.
- Chakravarty, S. The logic of investment planning. (Contributions to Economic Analysis, XVIII.) Amsterdam, North-Holland, 1959. 12 + 170 pp. \$4.00.
- Chernoff, H., and Moses, L. E. Elementary decision theory. New York, Wiley, 1959. 15 + 364 pp. \$7.50.
- Dodge, H. F., and Romig, H. G. Sampling inspection tables: Single and double sampling. 2d ed., rev. and expanded. (A Wiley Publication in Applied Statistics.) New York, Wiley, 1959. 11 + 224 pp.
- Evanusa, S. See Gantmacher, F. R.
- Freund, J. See Reichenbach, H.
- Gantmacher, F. R. Applications of the theory of matrices. Trans. by J. L. Brenner, with the assistance of D. W. Bushaw and S. Evanusa. New York, Interscience, 1959. 9 + 317 pp. \$9.00.
- Gordevskii, D. Z. Zadači po analitičeskoj geometrii na obrazovanie liniĭ i poverhnosteĭ. Kharkov, Izdat. Har'kovsk. Gos. Univ. im A. M. Gor'kiĭ, 1958. 51 pp. 1.40 rubles.
- Gutenberg, E. See Beckmann, M. J.
- Hecke, E. Mathematische Werke. Ed. on commission by the Akademie der Wissenschaften zu Göttingen. Göttingen, Vandenhoeck and Ruprecht, 1959. 955 pp., 1 plate. 70.00 DM.
- Kondo, K. See Raag memoirs.
- Logičeskie issledovaniya. Sbornik statei. Moscow, Izdat. Akad. Nauk SSSR, 1959. 466 pp. 18.50 rubles.
- Moses, L. E. See Chernoff, H.

- Peano, G. *Opere scelte*. Vol. III. *Geometria e fondamenti, Meccanica razionale, Varie*. Ed. by the Unione Matematica Italiana with contributions by the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1959. 7 + 470 pp. 5000 Lire.
- Pérès, J. See Boscher, J.
- RAAG memoirs of the unifying study of basic problems in engineering and physical sciences by means of geometry. Vol. II. Ed. by K. Kondo. Tokyo, Gakujutsu Bunken Fukyu-Kai, 1958. 16 + 589 pp., 4 plates. \$17.50.
- Raschewski, P. K. *Riemannsche Geometrie und Tensoranalysis*. (Hochschulbücher für Mathematik, Bd. 42.) Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 606 pp. 42.00 DM.
- Reichenbach, H. *The philosophy of space and time*. Trans. by M. Reichenbach and J. Freund, with introductory remarks by R. Carnap. New York, Dover, 1958. 16 + 295 pp. \$2.00.
- Reichenbach, M. See Reichenbach, H.
- Romig, H. G. See Dodge, H. F.
- Samarski, A. A. See Tychonoff, A. N.
- Schwartz, L. *Matematica y fisica cuantica*. Universidad de Buenos Aires, Facultad de Ciencias Exactas y Naturales, Departamento de Matematicas, 1958. 3 + 266 pp., mimeographed.
- Severi, F. *Geometria dei sistemi algebrici sopra una superficie e sopra una varietà algebrica*. Rome, Cremonese, Vol. 2, 1958, 4 + 464 pp., 4500 Lire; Vol. 3, 1959, 8 + 447 pp., 4800 Lire.
- Szegö, G. *Orthogonal polynomials*. Rev. ed. (American Mathematical Society Colloquium Publications, Vol. 23.) Providence, American Mathematical Society, 1959. 9 + 421 pp. \$10.60.
- Trachtenbrot, B. A. *Wieso können Automaten rechnen?* Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 101 pp. Paperbound, 3.60 DM.
- Tychonoff, A. N., and Samarski, A. A. *Differentialgleichungen der mathematischen Physik*. (Hochschulbücher für Mathematik, Bd. 39.) Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 660 pp. 42.00 DM.
- Zimmermann, K. F. *Tabellen, Formeln und Fachausdrücke zur Variationsstatistik: Für Landwirtschaftswissenschaftler, Naturwissenschaftler, Mediziner und Ingenieure*. Berlin, VEB Deutscher Verlag der Wissenschaften, 1959. 129 pp. 12.80 DM.

# ABSTRACTS OF CONTRIBUTED PAPERS

THE FEBRUARY MEETING IN NEW YORK, NEW YORK

February 28, 1959

554-37. Henry Teicher: A question concerning positive-type polynomials.

A polynomial is of positive-type if all its coefficients are non-negative. For  $\epsilon > 1/4$ , in answer to Slepian (Bull. Amer. Math. Soc. vol. 64 (1958) p.59) the smallest integer  $N(\epsilon)$  is determined for which there exists a polynomial  $P_N(x)$  with all roots real, such that  $(x^2 - x + \epsilon) P_N(x)$  is of positive type. Further,  $P_N(x)$  may be chosen to have equal roots (without increasing  $N$ ) and for a special sequence of  $\epsilon$  values must be so chosen. The method is elementary and consists in showing that a necessary condition for root reality is sufficient to resolve the problem. (Received January 16, 1959.)

554-38. H. F. Weinberger: Lower bounds for eigenvalues.

Let  $A$  be a linear operator defined on a (not necessarily closed or dense) subspace  $V$  of a Hilbert space  $H$ . Let its eigenvalues be defined by  $\lambda_k = \sup_{\phi_1 \in H} \inf_{u \in V; (u, \phi_1) = \dots = (u, \phi_{k-1}) = 0} (Au, u) / (u, u)$ . If elements  $p_1, \dots, p_N \in H$  and a constant  $\rho_N$  are given such that  $(Av, v) \geq \rho_N (v, v)$  for all  $v \in V$  such that  $(v, p_1) = \dots = (v, p_N) = 0$ , lower bounds for the eigenvalues of  $A$  are found in the following way. Let  $q_1, \dots, q_M$  be arbitrary elements of  $V$  and let the matrix  $(q_\alpha, p_i)$  be of rank  $R$ . Let  $\mu_1 \leq \dots \leq \mu_R$  be the first  $R$  roots of the polynomial equation  $\det\{(\rho_N - \mu) [(Aq_\alpha, q_\beta) - \mu(q_\alpha, q_\beta)] - (\bar{P}^{(N)} [Aq_\alpha - \mu q_\alpha], Aq_\beta - \mu q_\beta)\} = 0$  where  $\bar{P}^{(N)}$  is the orthogonal projection from  $H$  into the closure of the set of elements of  $V$  orthogonal to  $p_1, \dots, p_N$ . Then  $\lambda_{k+N-R} \geq \mu_k$ ,  $k = 1, \dots, R$ . This result improves and generalizes the bounds of Temple and T. Kato and their generalization by N. J. Lehmann and H. J. Maehly as well as those given by the author, W. Borsch-Supan, and F. Koehler. (Received January 16, 1959.)

554-39. Pinchas Mendelson: An odd dimensional center.

This paper grew out of a suggestion by L. Markus. H. Teraska (Jap. Jour. of Math. vol. 14 (1938) pp. 1-13) constructed a family  $F$  of curves in  $E^3$  with the following properties: (i) given any point  $P \in E^3$  there exists one and only one curve of the family which passes through  $P$ ; (ii)  $F$  is regular in the sense of H. Whitney [see H. Whitney, Regular families of curves, Ann of Math. vol. 34

(1933)]. A suitable transformation of  $E^3$  yields a new family  $F'$ , which is employed to define a dynamical system  $D$  throughout  $E^3$ .  $D$  has the following attributes: the origin  $0$  is the only critical point of  $D$ ; every orbit of  $D$  is bounded (hence Lagrange stable); there exist "many" periodic motions in arbitrarily small neighborhoods of  $0$ ; every orbit is bounded away from  $0$ , i.e.  $0$  is neither in the  $\alpha$ - nor in the  $\omega$ - limit set of any motion. The construction generalizes easily to  $E^{2n+1}$ . (Received January 19, 1959.)

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559-187. R. N. Snow: N-dimensional age dependent branching processes.

Preliminary report. I. Formulation.

The one-dimensional age-dependent model of Bellman and Harris is generalized and analogous results are obtained. An object of type  $j$ ,  $j = 1, 2, \dots, N$ , of age 0, has a life-length  $y$  with a probability distribution  $g_j(y)$ ; at the end of its life-length, it transforms, producing  $r_1, r_2, \dots, r_N$  progeny according to the probability functions  $\pi_j(y, r)$ . The progeny transform in turn, producing a family tree of objects. A measure is established on the space of family trees thru the defining functions  $g_j$  and  $\pi_j$ . The vector random variables  $z_j(t)$ :  $z_{j1}(t), z_{j2}(t), \dots, z_{jN}(t)$ , representing the number of objects alive at time  $t$  in the family tree originating from a type  $j$  object, are defined on this space. For  $s$  a complex vector,  $|s_j| \leq 1$ , let  $f_j(s, t) = E[\exp(z_j(t) \cdot \log s)]$ ,  $a_j(y, s) = \sum_r \pi_j(y, r) \exp(r \cdot \log s)$ ,  $a_{jk}(y) = (\partial / \partial s_k) a_j(y, s) |_{s=1}$  and  $H(t) : \{ \int_0^t dh_{jk}(y) \} = \{ \int_0^t a_{jk}(y) dg_j(y) \}$ . For  $H(\infty) < \infty$ , the generating functions  $f_j(s, t)$  are the unique solutions bounded by 1 of the equations  $f_j(s, t) = s_j(1 - g_j(t)) + \int_0^t a_j(y, f(s, t - y)) dg_j(y)$ . The mean values  $E[z_{jk}(t)] = m_{jk}(t)$  are functions of bounded variation satisfying the renewal type equations  $m_{jk}(t) = \delta_{jk}(1 - g_j(t)) + \int_0^t \sum_i m_{ik}(t - y) dh_{ji}(y)$ . Finally processes are classified as mutational or irreducible according to whether the matrix  $H(\infty)$  is decomposable or indecomposable. (Received October 5, 1959.)

559-188. R. N. Snow: N-dimensional age dependent branching processes.

Preliminary report, II. Asymptotic behavior, irreducible case.

The asymptotic behavior of the vector random variables  $z_j(t)$  is investigated for irreducible processes. The latter are categorized thru the extinction probabilities  $q_j$  and the maximum real characteristic root  $\lambda$  of the matrix  $H(\infty)$  into three disjoint classes: (a) degenerate ( $q_j = 1, \lambda \leq 1$ ), (b) singular ( $q_j = 0, \lambda = 1$ ) and (c) growth ( $q_j < 1, \lambda > 1$ ). For a growth process, let  $a$  be the root of largest real part of the determinantal equation  $\text{Det}(I - U(z)) = 0$  where  $U(z) = \{ \int_0^\infty \exp(-zy) dh_{jk}(y) \}$ . Then  $z = a$  is a simple root; correspondingly there exist positive right and left eigenvectors  $\mu, \eta$  of  $U(a)$ , unique up to a constant. The mean values  $m_{jk}(t)$  have the asymptotic behavior  $m_{jk}(t) \sim \exp(at)(c\mu_j v_k)$  as  $t \rightarrow \infty$ , where  $v_k = \eta_k(1 - \int_0^\infty \exp(-ay) dg_k(y))$ . If, in addition,  $a_{ijk}(y)$

$= (\partial^2 / \partial s_j \partial s_k) a_1(y, s) |_{s=1} < \infty$ , then the mixed moments  $E[z_{ij}(t)z_{ik}(t)]$   
 $\sim \exp(2at)(c_i \mu_j \nu_k)$  as  $t \rightarrow \infty$ . Under quite general conditions on the  $g_k$ , the ran-  
 dom variables  $w_j(t) = z_j(t) \exp(- at)$  converge in mean square to random vari-  
 ables  $w_j$  as  $t \rightarrow \infty$ . The components of each  $w_j$  are perfectly correlated, with  
 common distribution functions  $k_j(x)$ . The moment generating functions  $\phi(z)$  of  
 the  $k_j(x)$  satisfy the equations  $\phi_j(z) = \int a_j(y, \phi(z \exp(- ay))) dg_j(y)$ . Except for  
 jumps of magnitude  $q_j$  at  $x = 0$ , the  $k_j(x)$  are continuous and absolutely continu-  
 ous if  $1 - g_j(t) = O(\exp(- cy))$ . (Received October 5, 1959.)

October 31, 1959

560-41. Ralph Abraham: Jump conditions in Lichnerowicz space-time.

Algebraic conditions on the jump discontinuities of the Einstein tensor,  $G_{jk}$ , and its first covariant derivatives across a unique segment of a discontinuity hypersurface,  $\Sigma$ , in Lichnerowicz space-time are considered. The O'Brien-Syngé jump conditions,  $[G_{jk}]N^k = 0$ , where  $N_k$  is the unit normal to  $\Sigma$ , and the brackets indicate the jump across  $\Sigma$ , known since 1952 for general relativity, have been established by Isreal (Proc. Roy. Soc. A., no. 1254, vol. 248, p. 404) in the framework of Lichnerowicz space-time. In this paper, the method of Isreal is used to examine the characteristic jump conditions  $[\nabla_k G_{rs}]N^s = 0$ . It is shown that the characteristic conditions hold if and only if the discontinuities are uniform in the sense that  $[\nabla_k G_{rs}]B^k = 0$ , for all  $B^k$  tangent to  $\Sigma$ . Further, for characteristic manifolds,  $[G_{rs}] = 0$ , it is shown that  $[\nabla_k G_{rs}]N^s = \sigma[\nabla_s G_r^s]N_k$ , where  $\sigma$  is the orientation indicatrix of  $\Sigma$ , so that the characteristic conditions provide only four nontrivial jump relations, those obtained by taking the jump of the conservation equations, when  $[\nabla_k G_{rs}]$  is of the form  $N_k A_{rs}$ . (Received August 24, 1959.)

560-42. J. R. Büchi: Regular canonical systems and finite automata.

A Post canonical system is called regular if all its productions are of the form  $a_1x, \dots, a_nx \rightarrow bx$ . Some letters in the alphabet of a system are designated, and  $T(\Sigma)$  is the set of all words which are theorems of  $\Sigma$ , and do not contain designated letters. A finite automaton (with binary output) may be defined to be a regular system of very special type, the designated letters are the states, the remaining letters are input states. An input-word  $x$  is accepted by the f.a.  $\Phi$  just in case  $x \in T(\Phi)$ . Theorem: For any regular system  $\Sigma$  one can construct a f.a.  $\Phi$  such that  $T(\Phi) = T(\Sigma)$ , i.e.,  $T(\Sigma)$  is a regular set of words. This extends two results of Rabin and Scott, I. B. M. Journal, April, 1959. Namely the ones on "non-deterministic f.a." and on "two-way non erasing a." (Received September 10, 1959.)

560-43. Eckford Cohen: Some sets of integers related to the k-free integers.

Let the positive integer  $n$  have distinct prime factors,  $p_1, \dots, p_t$ ,  $n = p_1^{e_1} \dots p_t^{e_t}$ . Denote by  $Q_k^*$  the set of integers  $n$  with  $e_i \not\equiv 0 \pmod{k}$ ,  $1 \leq i \leq t$ , where  $k \geq 2$ ; also let  $Q_k^*(x)$  denote the number of positive integers  $\leq x$  contained in  $Q_k^*$ . For simplicity let  $k = 2$ . In a previous paper (Abstract 558-29 Notices Amer. Math. Soc. vol. 6 (1959) p. 380), it was shown by a method analogous to that used in treating the number  $Q_2(x)$  of square-free integers  $\leq x$ , that  $Q_2^*(x) = \alpha x + O(x^{1/2} \log x)$ , where  $\alpha = (\pi^2/6) \prod (1 - 2p^{-2} + p^{-3})$ . In this paper, by a method which reduces the consideration of  $Q_2^*(x)$  directly to that of  $Q_2(x)$ , it is shown that  $Q_2^*(x) = \alpha x + O(x^{1/2})$ . While the new method is elementary, it involves the use of generating functions. The paper treats the general case along the lines outlined for  $k = 2$ . Some related questions are also considered. (Received September 17, 1959.)

560-44. Herbert Federer: On certain measures associated with continuous maps of finite area. III.

In continuation of the work reported in Abstracts 558-33 and 558-34 Notices Amer. Math. Soc. vol. 6 (1959) pp. 381-382, it has now been proved that  $|\Psi_f|(M)$  equals the Lebesgue area of  $f$ . (Received August 3, 1959.)

560-45. Solomon Feferman: Nonrecursiveness of sets of sentences closed under direct product.

Let  $L$  be a first-order functional calculus with identity, and with a recursive syntax. Let  $P_{i,j}(L)$  be the set of sentences of  $L$  which are, for  $i = 0$ , closed under direct product, and, for  $i = 1$ , closed under direct power of the following number of factors: for  $j = 0$ , arbitrarily many, for  $j = 1$ , finitely many, and for  $j = 2$ , two. Let  $S_k(L)$  be, for  $k = 0$ , the set of all sentences of  $L$ , for  $k = 1$ , the set of all sentences of  $L$  in prenex form, and for  $k \geq 2$ , the set of all prenex sentences of  $L$  with at most  $k$  interchanges of existential and universal quantifiers in the prefix. A. Oberschelp (Archiv Math. Logik Grundlagenforsch vol. 4 (1958) pp. 95-123) has shown that  $P_{0,0}(L)$  is recursively enumerable; he asked whether this set is recursive. This is answered here in the negative, as a special case of the following theorem: for suitable choice of  $L$ , none of the sets  $P_{i,j}(L) \cap S_k(L)$  is recursive. The proof is based on the construction, for any recursively enumerable set  $A$ , of a sequence of sentences

$\phi_n \in S_2(L)$  (with quantifiers in either order) such that if  $n \in A$  then  $\phi_n$  is closed under arbitrary direct product, and if  $n \notin A$  then  $\phi_n$  is not closed under direct square. (Received August 24, 1959.)

560-46. R. R. Goldberg: Convolutions and Watson transforms.

The following result holds for any Watson transform: If  $f$  and  $g$  are transforms of each other in  $L^2(0, \infty)$  then so are  $\int_0^\infty x^{-1} \phi(y/x) f(x) dx$  and  $x^{-1} \int_0^\infty \phi(y/x) g(y) dy$ . Here  $\phi$  is any function satisfying  $\int_0^\infty |\phi(t)| t^{-1/2} dt < \infty$ . (The special case in which the Watson transform is the Fourier cosine transform and  $\phi$  is the characteristic function of  $(0, 1)$  is due to Titchmarsh.) An easy consequence is the following theorem of Bochner and Chandrasekharan: If  $k$  is a Watson transform,  $k: f \rightarrow g$ , then the map  $e^{-x/2} f(e^{-x}) \rightarrow e^{x/2} g(e^x)$  is a unitary operator on  $L^2(-\infty, \infty)$  which commutes with translations. (Received July 17, 1959.)

560-47. S. W. Golomb: The expected length of the longest cycle.

A random permutation of  $N$  objects places the  $N$  objects on cycles. By considering all  $N!$  cases, it is possible to compute the expected length  $L_N$  of the longest cycle. It is proved that the sequence  $\{L_N/N\}$  tends to a limit  $\lambda$  as  $N \rightarrow \infty$ . It is quite simple to prove that  $1/2 \leq \lambda \leq 2^{1/2}/2$ . A deeper argument shows that  $\lambda \leq 1 - (1/e)$ . Refinements of this technique show that  $.61003... < \lambda < .62891...$ . A computer program was used to obtain the result  $\lambda = .62432965...$ . In particular,  $\lambda$  is not equal to  $1 - (1/e)$ , nor  $5/8$ , nor  $(5^{1/2} - 1)/2$ , nor  $(\pi/8)^{1/2}$ . One is tempted to conjecture that  $\lambda$  is a new universal constant. The error term in the approximation of  $L_N/N$  to  $\lambda$  is defined by  $e_N = L_N - \lambda N$ . It is not even known that the sequence  $\{e_N\}$  is bounded; but there is some evidence for conjecturing that  $\{e_N\}$  converges, and that the limit is  $\lambda/2$ . (Received September 3, 1959.)

560-48. Bernard Grünbaum: Some theorems of Helly's type.

The following modification of Helly's theorem on intersections of convex sets is established, together with some related results. Let  $K$ , a subset of a Euclidean space, be the union of a finite number of disjoint, compact, convex sets. Then there exists an integer  $N = N(K)$  with the following property: For any family  $\{K_\alpha\}$ , where each  $K_\alpha$  is the transform of  $K$  by a nonsingular affine transformation  $T_\alpha$  (depending on  $\alpha$ ), if any  $N$  of the sets  $K_\alpha$  have a nonempty

intersection, then the intersection of all the sets  $K_{\alpha}$  is not empty. Examples show that none of the underlined conditions may be dropped, and that  $N(K)$  is not bounded even for subsets  $K$  of the real line. (Received September 8, 1959.)

560-49. E. O. A. Kreyszig: Classes of harmonic functions of three variables.

There exist various integral operators  $w = P(f)$  transforming complex analytic functions  $f$  into solutions  $w(x,y,z)$  of the Laplace equation  $\Delta w = 0$  in three variables. By using these operators the theory of the solutions  $w$  can be developed on the base of the theory of analytic functions. In this connection only those operators which preserve various basic properties of  $f$ , are of practical interest. Taking the operator  $w = P(f) = (2\pi i)^{-1} \int_{|t|=1} f dt$  [where  $f$  is an analytic function of  $t$  and  $u = x + (z + iy)t/2 - (z - iy)/2t$ ] introduced and considered by Whittaker and Bergman, and choosing different classes of associated functions  $f$  one obtains a classification of harmonic functions  $w$  such that the functions of each class have certain basic properties in common. For example, if  $f(u,t) = [p(u,t)]^{-1}$  where  $p$  is a polynomial of  $u$  and  $t$  the corresponding harmonic functions are singular on algebraic curves of maximum degree  $r \leq 2(n-1)[n/2]$  where  $n$  is the degree of  $p$  in  $t$ . If the coefficients of  $p$  are real some of the curves of singularities of  $w$  lie entirely in the planes  $y = 0$  or  $z = 0$ . Furthermore, several functions  $p$  may correspond to harmonic functions which are different but are singular on precisely the same curves in  $xyz$ -space. (Received March 12, 1959.)

560-50. E. O. A. Kreyszig: Bounds for amplitudes of limit cycles.

Let  $f(q)$  be an odd polynomial which is positive for all positive  $q$ . Let  $h(q)$  be an even polynomial of degree  $m \geq 2$  and such that  $h(q) \geq 0$  for  $|q| \leq \beta$  where  $\beta$  is some positive number. Then (1)  $\ddot{q} - (\text{sgn } \dot{q}) h\dot{q}^2/2 + f/2 = 0$  describes oscillations having a unique limit cycle. The corresponding maximum displacement  $Q^*$  ( $> 0$ ) is obtained as the positive root of  $I(q) = I(-q)$ . Here  $I(q) = \int_0^q a(x)f(x)dx$ ,  $a(x) = \exp[\int_0^x h(y)dy]$ . Furthermore  $I(q) > I(-q)$  for sufficiently small  $q > 0$ , and  $I(q) < I(-q)$  for sufficiently large  $q$ . Hence if  $J(q)$  is a function satisfying  $J(q) > I(q)$  and  $J(-q) < I(-q)$  for all  $q > 0$  and if the equation (2)  $J(q) = J(-q)$  has a positive root  $\bar{Q}$  then  $\bar{Q}$  is an upper bound for  $Q^*$ . For various types of functions  $f$  and  $h$  which are of practical interest, suitable functions  $J(q)$  can be obtained in a systematic manner, for example by considering

comparison problems, that is, by replacing  $h$  in (1) by a suitable function  $H$ . This procedure may lead to an equation (2) involving only known higher transcendental functions or only elementary functions. Lower bounds for  $Q^*$  can be derived in a similar manner. Using appropriate functions  $J(q)$  the bounds will be of relatively high degree of accuracy. The methods can be generalized and used in connection with other relaxation oscillations. (Received March 12, 1959.)

560-51. E. F. Moore: Minimality of complete relay decoding networks.

A complete relay decoding network is a network of contacts of  $n$  relays, which, as these relays assume each of their  $2^n$  possible states, connects a special node (called the root) to each of  $2^n$  other nodes (called leaves), one at a time. A decoding network has sneak paths if, for some state of the relays, two different leaves are connected together. The commonly constructed complete relay decoding network has no sneak paths, is called a tree, and uses  $2(2^n - 1)$  contacts. Lupanov showed (Dokl. Akad. Nauk SSSR vol. 119 (1958) pp. 23-26, English translation in Automation Express vol. 1 (1958) pp. 7-8) that complete relay decoding networks (which topologically are not trees in this instance) can be constructed which asymptotically use only  $2^n$  contacts, at the expense of letting them have sneak paths. Theorem: Any complete relay decoding network having no sneak paths uses at least  $2(2^n - 1)$  contacts. To prove this, consider the matrix having a row per state and a column per non-leaf network node, where each matrix entry is the name of the leaf connected to that node in that state. Since  $2^n$  names have at least  $2^n - 1$  occurrences each, the matrix must have at least  $2^n - 1$  columns. (Received June 23, 1959.)

560-52. E. F. Moore: Machine models of self-reproduction.

Von Neumann proposed two kinds of self-reproducing machines: early mechanical models using three-dimensional physical parts, and later tesselation models, with Euclidean space divided into cells, and time assuming only non-negative integer values. A configuration function associates with each of certain cells at one instant of time one of finitely many possible states. For time  $T > 0$  behavior is deterministic, a cell's state at  $T$  depending only on its neighbors' states at  $T - 1$ . A Garden-of-Eden configuration has finite domain and can occur only as initial ( $T = 0$ ) configuration, since no history can produce it. Self-reproducing configurations cannot have Garden-of-Eden subconfigurations. Theorem: Garden-of-Eden configurations can exist in any tesselation

universe in which configurations which can forget exist. Theorem: No self-reproducing configuration can be such that asymptotically its number of descendants is an exponential function of time. Although tessellation models exceed mechanical in amenability to formal mathematical manipulation, recently published papers by Penrose (Annals of Human Genetics (1958)), Jacobson (American Scientist (1958)) and Morowitz (ibid. (1959)) construct ingenious mechanical models which give various insights on the mathematical theory and on biological phenomena. (Received June 23, 1959.)

560-53. John Myhill: Real multiples of recursive equivalence types. I.

For background and notation see Bull. Amer. Math. Soc. vol. 64 (1958) pp. 373-376. In combinatorial analysis one often meets formulas of the form  $F(n)/G(n) \rightarrow \alpha$ , where  $F$  and  $G$  are recursive combinatorial (r.c.) functions and  $\alpha$  is an irrational number. This suggests trying to define, for at least some  $\alpha$  and  $X$ , the real multiple  $\alpha X$  in such a manner that  $F(n)/G(n) \rightarrow \alpha$  implies  $F(X) = \alpha G(X)$ . The method of Tarski, Cardinal algebras, pp. 38-39, is unavailable here, because it uses the fact that a monotone sequence possesses a least upper bound, which is true for elements of a cardinal algebra but conspicuously false for isols. Still worse, we can prove the Theorem. Let  $\alpha$  be a nonnegative weakly recursive real number (i.e.,  $\alpha = \lim f(n)/g(n)$  with  $f$  and  $g$  recursive; this is wider than the classical definition by e.g. J. Symb. Logic vol. 14 (1949) p. 155, Theorem IV); then there exist r.c. functions  $F_1, F_2, G$  such that  $F_1(n)/G(n)$  and  $F_2(n)/G(n)$  each converge to  $\alpha$  while yet for every isol  $X$ ,  $F_1(X) \neq F_2(X)$  (so that not both can be  $= \alpha G(X)$  however this is defined). (Received August 21, 1959.)

560-54. John Myhill: Real multiples of recursive equivalence types. II.

To remedy the situation described in the preceding abstract, we try defining  $A \approx \alpha B$  for R.E.T.'s  $A, B$  by:  $0 \leq p/q < \alpha$  implies  $pB < qA$  and  $p/q > \alpha$  implies  $pB > qA$ . Then we prove easily (for R.E.T.'s generally and not merely for isols) (i) the relation  $A \approx 1B$  is a congruence relation relative to  $+$  and  $\cdot$ , (ii) the equation  $A \approx \alpha B$  has at most one solution for  $\alpha$ , (iii)  $A \approx \alpha B$  implies  $B \approx A/\alpha$ , (iv)  $A \approx \alpha B$  and  $B \approx \beta C$  together imply  $A \approx (\alpha\beta)C$ . We still do not know whether for r.c. functions  $F, G$ ,  $F(n)/G(n) \rightarrow \alpha$  implies  $F(X) \approx \alpha G(X)$  for all infinite isols  $X$ ; the closest we have been able to get is Theorem I. Let  $F(n) = \sum c_i \binom{n}{i}$ ,  $G(n) = \sum c'_i \binom{n}{i}$  be r.c. functions; let  $c'_1 > 0$  and  $c_i/c'_i \rightarrow \alpha$ . Then (i)  $F(n)/G(n) \rightarrow \alpha$  and (ii)  $F(X) \approx \alpha G(X)$  for all infinite isols  $X$ . As easy

corollaries we obtain the Theorem of the preceding abstract and Theorem II. For every weakly recursive real number  $\alpha \geq 0$  there are infinite isols  $X, Y$  with  $X \approx \alpha Y$ ; more strongly, for every such  $\alpha$  there exist r.c. functions  $F, G$  such that  $F(X) \approx \alpha G(X)$  for all infinite isols  $X$ . We would like to thank J. C. E. Dekker, A. Nerode and V. L. Shapiro for ideas used in this and the preceding abstract. This research was supported by a grant from the Institute for Advanced Study, and by NSF grant G-7277. (Received August 21, 1959.)

560-55. N. C. Petridis: On quasiconformal mappings of several complex variables.

Definition: A quasiconformal mapping of the domain  $D$  of the complex plane into the unitary space  $V_n$  is a mapping  $f(z) = [f_1(z), \dots, f_n(z)]$  such that:  
 (1) each coordinate function is quasiconformal ( $C^1$  function of bounded dilatation),  
 (2)  $f_1\bar{z}/f_1z = f_2\bar{z}/f_2z = \dots = f_n\bar{z}/f_nz$  in  $D$  with the exception of isolated points.

Theorem 1: The class of mappings defined above is the largest class satisfying:

(1) Each coordinate function  $f_i(z)$  is quasiconformal, (2) the above property is invariant under holomorphic transformations in  $V_n$ . Theorem 2 (Uniformization theorem):

If  $z = \phi(\mathcal{G})$  is a one-one  $C^1$  bounded function on  $D$ ,  $F_i(\mathcal{G}) = f_i\phi(\mathcal{G})$ , and  $F_j(\mathcal{G})$  is an analytic function for some  $i$ , then all the functions  $F_j(\mathcal{G})$

$= f_j\phi(\mathcal{G})$  where  $f_j(z)$  ( $j = 1, 2, \dots, n$ ) are the coordinate functions of the mapping

in the above definition, are analytic. Remarks: (1) The existence of such a function  $z = \phi(\mathcal{G})$  follows from results of S. Stoilow. (2) From the above definition and theorems a theory of pseudoanalytic curves can be developed easily, analogous to the theory of analytic curves developed by H. and J. Weyl and L. Ahlfors. (3) Property 2 of the above definition can be repeated in a holomorphically invariant definition of quasiconformal mappings of several complex variables, which, in the case of one function is equivalent to that given by S. Hitotumatu (S. Hitotumatu: J. Math. Mech., January, 1959). (Received September 16, 1959.)

560-56. Moses Richardson: Block design games. Preliminary report.

A simple game is defined in an obvious way by any block design in which every pair of blocks intersects and of which the incidence matrix has no two equal row vectors. This extensive family of simple games, only a small part of which (especially, the plane and odd-dimensional finite projective games) have been previously investigated, is studied in this paper. Among the theorems

proved are the following. The game is not strong and has no main simple solution if the design is any symmetric balanced design with  $\lambda > 1$ , the dual of any balanced design with  $\lambda > 1$ , the dual of any partially balanced design with all  $\lambda_i > 1$ , any symmetric regular group divisible design with  $1 < \lambda_1 < \lambda_2$ , any symmetric semi-regular group divisible design with  $\lambda_1 > 1$ , etc. For balanced designs with  $\lambda = 1$ , the dual is not strong if it has  $k = 3$ ,  $r \geq 4$ , or  $k \geq 4$ ,  $r \geq 3$ ; this includes the nontrivial Steiner triple systems, and various designs obtained from finite euclidean and projective spaces including the even-dimensional finite projective games. The dual of the system of lines in any finite euclidean plane, and the dual of any nontrivial Steiner triple system have no equitable main simple solution. These results can be interpreted in terms of graphs and network flows. (Received August 11, 1959.)

560-57. J. B. Rosen: Stability of nonlinear system of difference and differential equations.

The system of  $n$  nonlinear differential equations (\*)  $dy/dt = -f(y,t)$ ,  $y \equiv \{x_1, x_2, \dots, x_n\}$ ,  $f \equiv \{f_1, f_2, \dots, f_n\}$ , is considered, with  $y(0) = y_0$ . Denote by  $B(y,t) \equiv (\partial f_i / \partial x_j)$  the  $n \times n$  first derivative matrix of  $f(y,t)$ . It is assumed that  $f(0,t) = 0$ , and that  $B(y,t) + B^T(y,t)$  is positive semi-definite for  $\|y\| \leq \|y_0\|$  and  $t \geq 0$ . Let  $y_j$  be the solution of the implicit finite difference approximation to (\*),  $y_{j+1} - y_j = -2^{-1} \Delta t (f_{j+1} + f_j)$ ,  $j = 1, 2, \dots, \infty$ ,  $f_j \equiv f(y_j, j \Delta t)$ . Then,  $\|y_j\| \leq \|y_0\|$ ,  $j = 1, 2, \dots, \infty$ , for every  $\Delta t > 0$ . It is also shown that  $y_j$  converges, as  $\Delta t \rightarrow 0$ , to the solution of (\*) with an error of  $O(\overline{\Delta t}^2)$  for  $0 \leq t \leq T$ . The solution of (\*) therefore satisfies  $\|y(t)\| \leq \|y_0\|$  for  $0 \leq t \leq T$ . (Received August 24, 1959.)

560-58. E. R. Suryanarayan: Intrinsic form of the relativistic equations of motion.

Consider a family of  $\omega^2$  world lines forming a three dimensional hypersurface,  $V_3$ , in the space-time of general relativity,  $V_4$ , spanned by a general coordinate system  $x^j$ : ( $j = 0, 1, 2, 3$ ). At each point of  $V_3$ , introduce four mutually orthogonal unit vectors; one of these vectors  $\lambda^j$  is tangent to the world line, another  $n^j$  normal to  $V_3$  and the remaining two  $a^j$  and  $b^j$  lie in  $V_3$ . By the use of stress energy equations, the directional derivatives of pressure,  $p$ , and the generalized density  $\eta$  in each of these directions are expressed in terms of the geodesic and normal curvatures of the world line and a scalar  $M$  which is the

algebraic sum of the principal values of the symmetric part of the covariant derivative of  $\lambda_j$ . Also from the conservation of mass, the directional derivative of the density,  $\rho$ , along the world line direction is expressed in terms of  $M$ . By eliminating  $M$  between two of these equations and using the first law of thermodynamics, a result obtained by Taub on wave speed (Relativistic Rankine-Hugoniot equations, Physical Review vol. 74 (1948) pp. 328-334) for isentropic fluids in the special relativity case is generalized to the case of nonisentropic fluids in general relativity. Again, if there exists a family of  $\omega^1$  hypersurfaces orthogonal to  $\lambda^j$  then these surfaces are minimal if and only if  $\rho$  is constant along each world line; and the motion is irrational if and only if  $\rho$  is constant along this family. (Received August 27, 1959.)

560-59. Felix Tan: Generalized Prandtl-Meyer flows.

The class of fluid motions to be studied here is characterized by possessing one family of  $\omega^1$  plane characteristic surfaces tangent to a cylindrical surface. Considerations will be limited to the case where the bicharacteristics and their orthogonal trajectories are two families of parallel straight lines in each characteristic plane. An additional assumption is also made that the stagnation entropy and the specific enthalpy are both constant. The well-known two-dimensional Prandtl-Meyer flows are flows that possess straight line bicharacteristics. It will be shown that the class of fluid motions under consideration are a generalization of the Prandtl-Meyer flows in the sense that: (1) the present flows are space flows; (2) the angle  $\bar{\phi}$ , which the bicharacteristics make with the  $z$ -axis varies from one characteristic plane to another (in the Prandtl-Meyer flows,  $\bar{\phi} = \pi/2$ ); (3) the flows are irrational. (Received August 27, 1959.)

560-60: E. F. Whittlesey: The fundamental group and homology groups of a class of 2-complexes.

Let  $K$  be a connected finite complex which is at most 2-dimensional and wherein every 1-cell is of degree at most two. Theorem: The fundamental group of  $K$  is the free product of the groups of the closed 2-manifolds corresponding to its closed surface components and  $1 + s_c - \chi - q_c - 2p_c$  free cyclic groups, where  $s_c$  is the number of closed surface components, and  $p_c$  and  $q_c$  are, respectively, the sums of the genera of the orientable and nonorientable closed surface components. Theorem: If  $p_i$  is the  $i$ th Betti number of  $K$ , and

$s_0(s_n)$  the number of (non) orientable closed surface components, then ( $p_0 = 1$ ),  $\chi = 1 + s_0 - X$ ,  $p_2 = s_0$ ; there are  $s_n$  torsion coefficients = 2 in the dimension  $n$ , and no others. These theorems unify the familiar results for 1-complexes and 2-manifolds and generalize them to include 2-pseudomanifolds with or without attached linear graph. The results follow directly from the canonical form for a 2-complex. (Received June 8, 1959.)

560-61. E. F. Whittlesey: The Königsberg theorem in two dimensions. I

Let  $K$  be a connected finite 2-complex. For simplicity, suppose  $K$  is not disconnected by the removal of its vertices. Polygonal or 2-paths in  $K$  are formed by pasting polygons of  $K$  edge to edge. A 2-path is unicursal if no (unoriented) polygon of  $K$  occurs in it more than once. A set of 2-paths are disjoint if no two have an (unoriented) polygon in common. A set of disjoint, unicursal 2-paths, minimal in number, covering  $K$  (a minimal unicursal covering) is determined algorithmically. The problem is combinatorial, not topological, in the sense that a minimal unicursal covering of  $K$  consists of just one 2-path provided only that each line component which lies on two or more surface components is split into at least two edges. (Received June 12, 1959.)

560-62. E. F. Whittlesey: The Königsberg theorem in two dimensions. II. Closed 2-paths.

Let  $K$  be a finite 2-complex which is not disconnected by the removal of its vertices. A 2-path in  $K$  is called closed mod 2 (mod 0) if it is the image of a closed (orientable) 2-manifold. Then it follows that a 2-path is closed mod 2  $\Leftrightarrow$  (regarded as a 2-chain) it is a 2-cycle mod 2  $\Leftrightarrow$  the degree of every edge in the 2-path is even; it is closed mod 0 iff, when its constituent polygons are coherently oriented, it is an integral 2-cycle. A minimal unicursal covering of  $K$  is called closed mod 2 (mod 0) iff the constituent 2-paths are all closed mod 2 (mod 0). There may be several such paths in a minimal unicursal closed covering. Theorem: A minimal unicursal covering of  $K$  is closed mod 2  $\Leftrightarrow$  the degree of every edge in  $K$  is even  $\Leftrightarrow \sum \sigma_i$  is a 2-cycle mod 2 (summation over all 2-cells of  $K$ ). Theorem: A minimal unicursal covering of  $K$  is closed mod 0  $\Leftrightarrow \sum \sigma_i$  is an integral 2-cycle, for suitable orientations of the 2-cells  $\sigma_i$ . (Received June 12, 1959.)

560-63. Jacob Wolfowitz: The capacity of a channel in which the probabilities of error are known only to the receiver or only to the sender.

Let  $\{w(\cdot|s), s \in S\}$  be a set, finite or infinite, of channel probability functions (c.p.f.'s); i.e.,  $w(j|i|s)$  is the probability that the symbol  $j$  is received when the symbol  $i$  is sent over a discrete memoryless channel, and  $s$  is an index used to distinguish the c.p.f.'s whose alphabets are the same. Suppose that the c.p.f. may change arbitrarily from one word of  $n$  symbols to the next, and, at the beginning of each word, is either (I) known to the receiver but not to the sender, (II) known to the sender but not to the receiver. For the stochastic input  $\pi$  let  $C(\pi|s) = H(\pi) - H'(\pi|s)$  be the difference of, respectively, the Shannon entropies of the input, and of the input, conditional upon the output, when  $w(\cdot|s)$  is the c.p.f. The capacity  $C$  of the channel is then, in Case I,  $C_1 = \sup_{\pi} \inf_{s \in S} C(\pi|s)$ , and in Case II,  $C_2 = \inf_{s \in S} \sup_{\pi} C(\pi|s)$ . (Thus knowledge of the c.p.f. by the receiver alone adds nothing to the capacity.)  $C_1 = 0$  implies  $C_2 = 0$ . Of course,  $C_1 \leq C_2$ . The coding theorem and strong converse are proved, both to within a factor  $\exp\{O(n^{1/2})\}$ . Let  $\epsilon > 0$  be arbitrary. There exist positive  $c_1$  and  $c_2$  with the following property: Let  $\lambda_n$  be the smallest probability of error among all codes with length  $2^{n(C-\epsilon)}$ . Then  $\lambda_n < c_1 e^{-c_2 n}$ . The results can be extended to certain semi-continuous channels and channels with memory. (Received June 15, 1959.)

November 20-21, 1959

561-1. H. S. Collins: Primitive idempotents in the semigroup of measures.

An idempotent  $\mu$  in a semigroup  $S$  with zero is primitive if the only idempotents in  $\mu S \mu$  are  $\mu$  and zero. This note investigates the existence of such idempotents in the convolution semigroup of measures  $S = S(G)$  on a compact group  $G$ . Theorem. Each of the following is equivalent for a nonzero idempotent  $\mu$  in  $S(G)$ : (1)  $\mu$  is primitive, (2)  $H = \text{carrier } \mu$  is a maximal proper closed subgroup of  $G$ , (3) either  $\mu$  is central and  $\mu S \mu|N = \mu G$  or  $\mu S \mu|N = \{\mu\}$ , where  $N$  denotes the set of nilpotent elements of  $S$ . If in addition  $\mu$  is central, each of the above is equivalent to each of (4)  $S\mu$  contains precisely two idempotents and (5)  $G/H$  is a finite cyclic group of prime order. Additional remarks are made concerning the two types of compact groups  $G$  encountered, namely those containing a maximal proper closed subgroup and those containing no proper closed subgroup save the identity subgroup. One result shows that a maximal proper closed subgroup is open if it is normal, while another proves that the second above mentioned type of group is necessarily cyclic of prime order. (Received August 24, 1959.)

561-2. J. G. Horne, Jr.: Countable paracompactness and cb-spaces.

A real valued function on a topological space  $X$  is locally bounded if each point has a neighborhood on which it is bounded. A function  $h$  is bounded in  $C(X)$  if there is  $f \in C(X)$  such that  $f \geq |h|$ . A topological space is a cb-space if every locally bounded function on  $X$  is bounded in  $C(X)$ . In general,  $\tilde{X}$  is a cb-space iff every upper semicontinuous  $h$  is bounded above in  $C(X)$ . There exist pseudo-compact, non-cb-spaces, and a cb-space need not be normal. The product of a compact space and a cb-space is a cb-space. If  $X$  is normal then  $X$  is a cb-space iff  $X$  is countably paracompact. Other characterizations of countable paracompactness are obtained, some of which extend the list of characterizations which are gotten from characterizations of paracompactness by appropriate insertion of "countable". Using one, a different proof of the countable paracompactness of linearly ordered spaces is given. Finally, it is shown that Sorgenfrey's example (the reals with the half-open interval topology) provides an example of a paracompact space (which is therefore a cb-space) whose pro-

duct with itself is neither countably paracompact nor a cb-space. (Received October 2, 1959.)

561-3. R. L. Plunkett: Openness of the derivative of a complex function.

A purely topological proof is presented of the theorem: If the complex valued function  $f$  has a nonconstant derivative in a region  $E$  of the complex plane  $Z$ , then the derivative,  $f'$ , is a strongly open mapping of  $E$  into  $Z$ . Also included are topological proofs of the following theorems which are used to obtain the above result: (1) If  $f$  is differentiable in  $E$ , then it is uniformly differentiable on each compact subset of  $E$ , and (2) If  $f'$  exists and is nonconstant on  $E$ , then  $(1/2\pi i)\mu_C(f', p) \cong 1$ , where  $C$  is any simple closed curve contained with its interior  $R$  in  $E$  and  $p$  is any point of  $f'(R \cup C) - f'(C)$ . Here  $\mu_C(f', p)$  is the topological index of  $f'$  on  $C$  relative to  $p$ . (See G. T. Whyburn, Topological analysis, Princeton University Press, 1958, Chapter V.) (Received September 23, 1959.)

561-4. Jack Segal: Homogeneity of the hyperspace of a continuum.  
Preliminary report.

In the New Scottish Book (p. 17) Knaster raised the question of the homogeneity of  $C(X)$  (the space of all subcontinua of a metric continuum  $X$  with the topology induced by the Hausdorff metric). In this paper it is shown that if  $X$  is not locally connected or is a linear graph, then  $C(X)$  is not homogeneous. So that if  $C(X)$  is finite dimensional it is not homogeneous. Also if  $X$  contains a linear graph which contains interior points relative to  $X$ , then  $C(X)$  is not homogeneous. (Received October 2, 1959.)

561-5. Leonard Carlitz: A note on exponential sums.

Let  $p$  be an odd prime and let  $\zeta$  denote a primitive  $p$ th root of 1. Put  $B = \sum_{s=1}^{p-1} c_s \zeta^s$ , where each  $c_s = \pm 1$ . Rédei (Acta Math. vol. 79 (1947) pp. 273-290) has proved that (\*)  $(1 - \zeta)^{(p-1)/2} | B$  if and only if  $B$  is a Gauss sum. If (\*) is not satisfied, then  $B$  is divisible by at most  $(1 - \zeta)^{(p-1)/4}$ ; this will occur if and only if  $p = 4m + 1$  and  $B$  is a certain combination of biquadratic Gauss sums. In the present paper the writer gives a new proof of Rédei's theorem and discusses some related results. For example  $|B|^2 \equiv 0 \pmod{p}$  if and only if  $B$  is a Gauss sum. If  $\sum c_s = 0$ , it is shown that the statement  $B = o(p)$  for all  $B$  is false; also the statement  $|B| > c > 0$ , for all  $B$ , where  $c$  is independent of  $p$ , is

false. If  $B_r = \sum_{s=1}^r c_s \zeta^{k_s}$ , where  $1 \leq k_1 < k_2 < \dots < k_r \equiv p-1$ , it is proved that  $|B_r|^2 \equiv 0 \pmod{p}$  is impossible for  $r < p-1$ ; also it is shown that for large  $p$  and  $r = o(p)$  one can find  $B_r$  with  $\sum c_s = 0$  such that  $|B_r| \sim r$ . Finally it is proved that  $(1-\zeta)^t |B|, (1-\zeta)^{t+1} \nmid B$  if and only if there exists a factorization  $x^{2m}-1 \equiv (x^m + a_t x^{m-t} + \dots + a_m)(x^m + b_t x^{m-t} + \dots + b_m) \pmod{p}$ , where  $p = 2m+1$  and  $1 \leq t \leq m$ . (Received October 5, 1959.)

561-6. M. H. Martin: A nonlinear boundary problem for harmonic functions.

If  $u_2$  is a nonconstant solution of the boundary problem  $\Delta u = 0$  in a region  $S$ ,  $u_n = h(s)u^{1+p}$  on the boundary  $C$  of  $S$ , harmonic in  $R \supset S$ , and if  $p$  is an even positive integer, no other such solution  $u_1$  exists for which the ratio  $\lambda = u_1/u_2$  is regular in  $R$  and  $|\lambda| < 1$ . This earlier result [Proc. Amer. Math. Soc. vol. 10 (1959) p. 264] of the author is extended to all  $p > 0$  under the more rigid requirement  $0 < \lambda < 1$  and rests on the determination of a more general solution to the controlling partial differential equation  $\sigma^2 \phi - 4(1+p)\rho\sigma\phi_\rho + 4(1+p)^2\sigma^2 = 0$  than had hitherto been obtained. (Received October 5, 1959.)

561-7. A. T. Brauer: Note on a number theoretical paper of Sierpinski.

W. Sierpinski [Collect. Math. vol. 10 (1958) pp. 81-83] has just published the following theorem: The set  $A$  of all primes which are divisors of integers of form  $2^x + 1$  contains all primes of the form  $8n \pm 3$  and infinitely many primes of the form  $8n + 1$ . The set  $B$  of all primes which are divisors of integers of the form  $2^{2s+1} - 1$  contains all primes of the form  $8n + 7$  and some primes of the form  $8n + 1$ . Every prime of form  $8n + 1$  belongs either to  $A$  or to  $B$ . The question whether the set  $B$  contains infinitely many primes of form  $8n + 1$  is raised, but remains open. In this note a simple proof of this result is given. Moreover, it is shown that  $B$  contains infinitely many primes of form  $8n + 1$ . (Received October 5, 1959.)

561-8. WITHDRAWN

561-9. P. D. Hill: Some sequences of groups for which an onto inverse limit is unique.

The limit of an inverse sequence of groups  $(G_n; h_n)$  is called an onto limit if the homomorphisms  $h_n$  of  $G_{n+1}$  into  $G_n$  are all onto. And, given a sequence

of groups  $G_n$ , an onto inverse limit of the sequence is said to be unique if it is independent of the choice of the onto homomorphisms. Let  $B(G)$  denote the number of elements in a basis of a finitely generated abelian group  $G$ . Theorem: An onto inverse limit of a sequence  $G_n$  of finitely generated abelian groups is unique if  $\{B(G_n)\}$  is bounded. (Received October 5, 1959.)

561-10. R. D. McWilliams: A note on weakly fundamental sequences in Banach spaces.

If  $X$  is a real Banach space, let  $J_X$  be the canonical mapping of  $X$  into  $X^{**}$ . If  $Y$  is a closed subspace of  $X$ , let  $K_Y(X)$  be the linear subset of  $X^{**}$  consisting of all  $x^{**}$  for which there is a sequence  $\{y_n\}$  in  $Y$  such that  $J_X y_n$  converges to  $x^{**}$  in the  $X^*$ -topology of  $X^{**}$ . Let  $K(X) = K_X(X)$ . Thus  $X$  is weakly sequentially complete if and only if  $K(X) = J_X(X)$ ; simple examples show that, in the relation  $J_X(X) \subseteq K(X) \subseteq X^{**}$ , the inclusions may or may not be proper. If  $X^*$  is separable, then  $K(X) = X^{**}$ . If  $Y$  is a closed subspace of  $X$  and if  $L$  is the mapping from  $Y^{**}$  into  $X^{**}$  defined by  $(Ly^{**})(x^*) = y^{**}(x^*|_Y)$  for all  $x^* \in X^*$ , then  $L$  is isometric, and  $L$  maps  $K(Y)$  onto  $K_Y(X)$ . If  $K(Y)$  is norm-closed in  $Y^{**}$  for each closed separable subspace  $Y$  of  $X$ , then  $K(X)$  is norm-closed in  $X^{**}$ . If  $Y$  is a subspace of  $X$  and there is a bounded projection  $P$  of  $X$  onto  $Y$ , then  $P^{**}$  maps  $K(X)$  onto  $K(Y)$ , and  $P^{**}$  and  $L^{-1}$  coincide on  $LY^{**}$ . (Received October 5, 1959.)

561-11. R. A. Struble: On the almost-periodic motion of an earth satellite. Preliminary report.

Modifying the analysis of King-Hele (Proc. Roy. Soc. London Ser. A, vol. 247 (1958) p. 49), the perturbation technique is shown to lead to a nonlinear, Mathieu-type equation rather than the traditional linear, simple-harmonic equation. A first order solution in the oblateness parameter then exhibits an almost-periodic motion in a precessing, near orbital plane which reflects the much discussed apsidal motion of the orbit. (Received October 5, 1959.)

561-12. C. H. Edwards, Jr.: Polyhedra in  $S^n$  whose complements have uniformly abelian local fundamental groups.

Suppose that  $C$  is a curved polyhedral complex imbedded in the  $n$ -sphere  $S^n$  such that (a)  $C = \bigcup_{i=1}^k D_i$ , where the  $D_i$ ,  $i = 1, \dots, k$ , are  $(n-1)$ -cell subcomplexes of  $C$  with disjoint interiors, and (b)  $D_j \cap \bigcup_{i=1}^{j-1} D_i$ ,  $j = 2, 3, \dots, k$ , is an

$(n - 2)$ -cell tamely imbedded in the  $(n - 2)$ - sphere  $Bd D_j$ . If  $S^n - C$  has uniformly abelian local fundamental groups, then  $\pi_1(S^n - C) = 1$  modulo the following conjecture: If  $p^{n-2}$  is a polyhedral  $(n - 2)$ -cell in  $S^{n-1}$ , then  $p^{n-2}$  is flat in  $S^{n-1}$ . (Observe that Guggenheim [Proc. London Math. Soc. vol. 3 (1952) pp. 129-152] has proved this conjecture for  $n \leq 5$ , and that the tameness hypothesis in (b) above is unnecessary for  $n \leq 5$ .) Generalizing a construction due to Harrold [Trans. Amer. Math. Soc. vol. 67 (1949) pp. 120-129], a subspace  $X = A \cup M$  of Hilbert space is defined such that (1)  $A$  is ULC<sup>1</sup> and homeomorphic with  $S^n - C$ , (2)  $M$  is an  $(n - 1)$ -sphere constituting the boundary in  $X$  of  $A$ , and (3)  $\pi_1(A, X) = 1$ . A theorem of Eilenberg and Wilder [Amer. J. Math. vol. 64 (1942) p. 620] then applies to give  $\pi_1(A) = 1$ , which implies  $\pi_1(S^n - C) = 1$ . (Received October 6, 1959.)

561-13. J. T. Moore: An extension of an early result of Stieltjes.

An early result of Stieltjes gives algebraic expressions, from which one can compute approximations for the remainders in the asymptotic expansions of Bessel functions of order 0. These results of Stieltjes have been improved, and also extended to include Bessel functions of order 1. (Received October 6, 1959.)

561-14. C. N. Moore: On an upper bound for the difference between consecutive primes.

The sieve recently developed by the author in order to prove the infinity of prime pairs, gives an exact formula for the number of primes in a given interval. It may therefore be used in discussing many other problems concerning the fine structure in the distribution of primes. In the present paper it is applied to showing that for all  $n$ , an upper bound for the difference  $p_{n+1} - p_n$ , where  $p_n$  represents the  $n$ th prime, is given by  $\log^2 p_n$ . (Received October 6, 1959.)

561-15. J. S. MacNerney: Operator-moment problems. Preliminary report.

Let  $S$  be a complex linear space,  $Q$  be an inner-product for  $S$ , and  $N$  be the corresponding norm —  $N(x) = Q(x, x)^{1/2}$  for  $x$  in  $S$ . Suppose that  $S$  is complete with respect to  $N$ , let  $T$  denote the set of all linear functions from  $S$  into  $S$  which are continuous with respect to  $N$ , and — for each  $A$  in  $T$  — let  $A^*$  denote the adjoint of  $A$  with respect to  $Q$  and  $P_A$  denote the projection from  $S$  onto the

closure with respect to  $N$  of  $A(S)$ . Theorem: If each of  $A$ ,  $B$ , and  $U$  is in  $T$  and there is a number  $m$  such that  $|Q(Ux,y)| \leq mN(Ax)N(By)$  for all  $x$  and  $y$  in  $S$ , there is only one  $V$  in  $T$  such that  $VP_A = P_B V = V$  and  $U = B*VA$ . This result, applied in spaces of sequences with values in  $S$ , is a connecting link between an extension of classic moment-problems on the real line (to corresponding problems with moments in  $T$ ) and ideas in the author's forthcoming paper, Investigation concerning positive definite continued fractions, Duke-Math. J. vol. 26 (December 1959). (Received October 6, 1959.)

561-16. R. A. Bradley and J. E. Jackson: Notes on sequential multivariate tests on means.

Multivariate tests of hypotheses on vectors of population means have long been available. Methods of sequential analysis were developed for military inspection systems. With the present sampling and inspection of complex military and civilian items, it is more and more important to have efficient multivariate inspection methods. Sequential methods result in minimum inspection of lots on the average. The paper develops the required sequential multivariate methods. Test statistics used are the  $T^2$  and  $X^2$  statistics used in fixed sample size methods applicable respectively when the population dispersion is known or unknown. The proper bounds for sequential inspection are obtained, tables have been prepared, and properties of the procedure discussed. Emphasis in the presentation is on mathematical aspects of the development of the methods. The research was sponsored by the Office of Naval Research through a contract with the Virginia Polytechnic Institute. (Received October 6, 1959.)

561-17. Leonard Carlitz and Jack Levine: Some problems concerning Kummer's congruences for the Euler numbers and polynomials.

Put  $e_m = c^m E_m(a)$ , where  $E_m(a)$  is the Euler polynomial of degree  $m$  and  $a, c$  are integral (mod  $p$ ),  $c \not\equiv 0 \pmod{p}$ . It is well known that if  $p$  is an odd prime,  $e_m$  satisfies (\*)  $\sum_{s=0}^r (-1)^s \binom{r}{s} e_{m+sw} \equiv 0 \pmod{(p^m, p^{rk})}$ , where  $p^{k-1}(p-1) | n$ . The present paper is concerned with the question of whether (\*) is best possible. It is proved for example that the least  $\mu$  such that  $e_{m+\mu} \equiv e_m \pmod{(p^m, p^k)}$  is  $\mu = p^{k-1}(p-1)$ , provided  $p > 3$ . It is also proved that the least  $t$  such that  $\sum_{s=0}^t (-1)^s \binom{t}{s} e_{m+sw} \equiv 0 \pmod{(p^m, p^{rk})}$  is  $t = r$  provided  $2^p \not\equiv 2 \pmod{p^2}$  and  $p \geq 2r^2 + 1$ . A special discussion of the case  $p = 3$  suggests that some of the general results may require modification for small values of  $p$ . (Received October 5, 1959.)

561-18. B. J. Boyer: Summability of derived Fourier series.

Let  $f$  be a  $C_k^p$  integrable function and let  $(\alpha, \beta)$  denote the Bosanquet-Linfoot summability scale. This paper is concerned with the summability of the  $r$ th derived Fourier series of  $f(x)$  ( $D_r F S f(x)$ ) in terms of the Fourier series of a function  $\omega$  closely related to  $f$ . The following results are established:

Theorem 1. If  $\omega$  is CP integrable and  $F S \omega(0) = 0(\alpha, \beta)$ , then  $D_r F S f(x) = a_r(\alpha + r, \beta)$ . Theorem 2. If  $D_r F S f(x) = a(\alpha + r, \beta)$ , then  $\omega$  is CP integrable and  $F S \omega(0) = 0(\alpha', \beta')$ . The  $(\alpha, \beta)$  and  $(\alpha', \beta')$  depend on  $f, \omega$  and  $r$ . The proofs of these results involve certain order relations and identities between the integral representations of  $D_r F S f(x)$  and  $F S \omega(0)$ . Theorem 2 relies heavily upon a general Tauberian result: If certain linear combinations of  $(\alpha + i, \beta + j)$  means sum a series which is summable (C), then the series was already summable  $(\alpha, \beta)$ . (Received October 7, 1959.)

561-19. M. K. Fort, Jr.: A lifting and neighborhood extension theorem for mappings into fiber spaces.

$Y$  is a locally trivial fiber space with base space  $B$ , fiber  $K$ , and projection  $p: Y \rightarrow B$ .  $A$  is a closed subset of a topological space  $X$ , and  $f: X \rightarrow B$  and  $\varphi: A \rightarrow Y$  are mappings such that  $p\varphi = f|_A$ . The theorem states: If  $K$  is an absolute neighborhood retract and  $X$  is paracompact, then there exists an open set  $N \supset A$  and a mapping  $\Psi: N \rightarrow Y$  such that  $\Psi|_A = \varphi$  and  $p\Psi = f|_N$ . (Received October 7, 1959.)

561-20. R. D. Anderson: Plane sections of topological spheres.

Let  $K$  be the set of all planes in  $E^3$  which are perpendicular to the  $Z$ -axis. There exists a topological 2-sphere  $S$  in  $E^3$  such that (1)  $S$  is tamely embedded and (2) for any  $k \in K$ ,  $k \cap S$  does not contain an arc. This result settles a question raised by Bing in his paper given at Salt Lake City. The larger question as to whether for any topological sphere in  $E^3$ , some plane section of it contains an arc is left open. The sphere  $S$  can be exhibited such that, either of two conditions obtains: (1) for  $k \in K$ ,  $k \cap S$  is a pseudo-arc or a circularly chainable circle of pseudo-arcs or (2) for  $k \in K$ ,  $k \cap S$  is hereditarily indecomposable. In the case of (1) the construction can be made so that each nondegenerate subcontinuum of  $k \cap S$  is homeomorphic to one of three continua: (a) a pseudo-arc, (b) a circularly chainable circle of pseudo-arcs or (c) a chainable arc of pseudo-arcs. (Received October 7, 1959.)

THE NOVEMBER MEETING IN LOS ANGELES, CALIFORNIA

November 21, 1959

562-1. M. A. Dengler: Analytical formulation of the physical properties of water and steam.

The evaluation of the performance and the heat rate of reheat or nonreheat steam electric power plants requires the analytical formulation of the properties of water vapor. Expressions for the specific volume  $v(T,p)$ , the enthalpy  $h(T,p)$  and the entropy  $s(T,p)$  of superheated or saturated steam are found in the steam tables of J. H. Keenan and F. G. Keyes. The aim of the present investigation was the derivation of mathematical expressions for further physical quantities known hereto only in tabular form. Equations, as deduced from principles of numerical analysis, have been established for: (a) the enthalpy  $h(\log p)$  of dry saturated steam, (b) the enthalpy  $h(T)$  of saturated water, (c) the enthalpy  $h(T,p)$  of compressed water, (d) the specific volume  $v(\log p)$  of saturated vapor, and (e) the enthalpies  $h_{(l)}(p)$ ,  $h_{(g)}(p)$  and the entropies  $s_{(l)}(p)$ ,  $s_{(g)}(p)$  of water and water vapor under saturation conditions. In addition to the formulation of the numerical approximations listed above an equation for the turbine state line has been derived. A code for the IBM 704 has been prepared and the program has been applied for performance analyses of existing power plants. The heat rate calculated was found to deviate less than two percent from experimental values. (Received June 15, 1959.)

562-2. L. W. Anderson and L. E. Ward, Jr.: A structure theorem for topological lattices.

Recently R. J. Koch proved that if  $X$  is a compact Hausdorff space with continuous partial order  $\leq$ , if  $L(x) = \{y: y \leq x\}$  is connected for each  $x \in X$ , and if  $X$  has a zero, then each element of  $X$  lies in some connected chain containing zero. This result fails if  $X$  is assumed to be locally compact and/or locally connected, or even a locally compact topological semi-lattice. It can, however, be extended in the following manner. If  $X$  is a connected and locally compact topological lattice and  $a \leq b$  in  $X$  then  $a$  and  $b$  lie in some connected chain. (Received August 17, 1959.)

562-3. L. W. Anderson and L. E. Ward, Jr.: One-dimensional topological semi-lattices.

A topological semi-lattice is a Hausdorff space  $S$  together with a partial order such that  $x \wedge y = \text{glb}(x,y)$  is a continuous function. A number of elementary results on such structures are obtained. The principal theorems are the following. Let  $S$  be a compact topological semi-lattice whose codimension does not exceed one. (I) If  $S$  is order-dense then  $S$  is a tree and the semi-lattice and cutpoint orderings are identical. (II) If  $S$  is connected then these statements are equivalent: (a) if  $x \in S$  then  $x \wedge S$  is a distributive lattice and  $\{z: y \leq z\} \cap (x \wedge S)$  is connected for each  $y \in x \wedge S$ , (b) if  $x \in S$  then  $x \wedge S$  is a chain, (c)  $S$  is order dense, (d)  $S$  is a tree and the semi-lattice and cutpoint orderings are identical. (III) If  $S$  is connected and locally connected then  $S$  is a tree. (Received August 17, 1959.)

562-4. G. J. Minty: Monotone networks.

Let  $G', G''$  be two closed subgroups of  $R$ .  $G' \times G''$  is partially ordered in the usual way. A resistor is a maximal simply-ordered subset of  $G' \times G''$ . A monotone network consists of a finite directed graph, a pair of such groups  $G', G''$ , and a function which assigns a resistor to each line of the graph. A solution is an element of the cycle-group (formed with  $G'$ ) and an element of the co-cycle group (formed with  $G''$ ) such that the element of  $G' \times G''$  assigned to each line lies in the resistor of that line. The following theorems hold: (1) If  $G' = G'' = R$ , the solutions are a closed convex set in  $R^n \times R^n$ ; (2) For each line of the graph, either the current or the voltage-drop is unique; (3) A solution exists if and only if the monotone network has no unbalanced co-cycle and no short-circuit, with appropriate definitions of these notions. The existence-proof for the case in which  $G'$  and  $G''$  are discrete furnishes a numerical method. Applications are given to electrical, mechanical, and other problems. The results may be considered a synthesis and extension of various results of Duffin, Millar, Ford, Fulkerson, Dantzig, and others, on electrical networks with monotone resistors and discrete network-flow problems. (Received August 26, 1959.)

562-5. R. M. Robinson: Arrangement of 24 points on a sphere.

The problem of arranging  $n$  points on a sphere so as to maximize the minimum distance between any two points has been solved previously only for  $n \leq 9$  and for  $n = 12$ . A proof is given here of the conjecture of van der Waerden con-

cerning the best arrangement of 24 points [Math. Ann. vol. 123 (1951) p. 123]. In the optimal arrangement, the points form the vertices of a system of 32 equilateral triangles and 6 squares on the sphere, with four triangles and one square meeting at each vertex. Let  $a$  be the edge of the triangles and squares (approximately  $43^\circ 41'$ ), which is the minimum distance between points. Let  $\Delta$  be the area of a triangle, and  $2\Delta'$  the area of a square. Then the required result is an easy consequence of the following lemma. Consider a saturated system of  $n$  points on the sphere with distances at least  $a$ . Then for a suitable division of the sphere into triangles with the  $n$  points as vertices, if  $k$  triangles meet at some vertex, then their combined area is at least  $4\Delta + (k - 4)\Delta'$ . Equality holds if and only if the triangles meeting at the point consist of four equilateral triangles and one or two half squares. (Received October 6, 1959.)

562-6. R. E. Haymond: Topologies for countably and finitely generated abelian groups.

Let  $J^n$  be the  $n$ -fold cartesian product of the set of rational integers with vector addition;  $\mathcal{S}_k$  ( $1 \leq k \leq n$ ) the family of subgroups of  $J^n$  with  $k$  basis elements. If  $(J^n, +, \mathcal{J})$  is a topological group and  $\mathcal{S}_k \subset \mathcal{J}$  (some  $k < n$ ) then  $\mathcal{J}$  is the discrete topology for  $J^n$ . If  $\mathcal{N}$  is the topology for  $J^n$  with  $\mathcal{S}_n$  as neighborhood system of  $\bar{0}$  then  $(J^n, +, \mathcal{N})$  is a nondiscrete metrizable  $T_4$  topological group. Other properties of this topology are derived and it is used to give topologies for any finitely generated abelian group. These topologies have similar properties and are discrete if and only if the group is finite. Analogous topologies are derived for countably generated abelian groups. (Received October 6, 1959.)

562-7. C. B. Bell, Jr., David Blackwell and Leo Breiman: A note on the completeness of order statistics.

A class,  $\Omega$ , of probability measures on an abstract measurable space  $(X, \mathcal{S})$  is said to be a symmetrically complete class for  $n = k$  (or a class for which the order statistic is complete) if for symmetric measurable  $h$  on  $(X^{(k)}, \mathcal{S}^{(k)})$   $\int h dF = 0$  for all  $F \in \Omega \implies h \equiv 0$  [F] for all  $F \in \Omega$ . Let  $\Omega_0(X)$ ,  $\Omega_1(X)$ ,  $\Omega_2(X)$  be, respectively, the classes of all probability measures; all non-degenerate probability measures; and all nonatomic probability measures on  $X$ . Further, if  $\lambda$  is a measure on  $\mathcal{S}$  and  $\mathcal{J} \subset \mathcal{S}$ , let  $\Omega_3(\lambda) = \{P | P \ll \lambda\}$  and  $\Omega(\mathcal{J}; \lambda) = \{\lambda_A | A \in \mathcal{J}; 0 < \lambda(A) < \infty; \lambda_A(C) = \lambda(AC)/\lambda(A), C \in \mathcal{S}\}$ . It is

proved that  $\Omega_0(X)$ ,  $\Omega_1(X)$ , and  $\Omega_2(X)$  are symmetrically complete for all  $n$ ; and that, if  $\lambda$  is a nonatomic  $\sigma$ -finite measure on  $\mathcal{A}$  and if semi-algebra  $\mathcal{A}$  generates  $\mathcal{L}$ , then  $\Omega(\mathcal{A}, \lambda)$ ,  $\Omega(\mathcal{A}, \lambda)$  and  $\Omega_3(\lambda)$  are symmetrically complete for all  $n$ . The bases of the proof are the facts that  $\Omega(\mathcal{A}, V)$  is symmetrically complete for properly chosen  $\mathcal{A} \subset \mathcal{L}$  and  $V$  (D. Fraser); the null classes of  $\Omega(\mathcal{A}, P_1)$  and  $\Omega_3(P_1)$  are equivalent; and  $\Omega_2(X) = \bigcup \Omega_3(P)$ ,  $P \in \Omega_2(X)$ . (Received October 6, 1959.)

562-8. C. E. Burgess: Continua which converge homeomorphically.

A sequence  $\{M_i\}$  of continua in a metric space is said to converge homeomorphically to a continuum  $M$  if for each positive number  $\epsilon$  there is an integer  $n$  such that if  $i > n$ , there is a homeomorphism of  $M_i$  onto  $M$  that moves no point more than  $\epsilon$ . Let  $C$  denote the class of all compact continua  $M$  such that there exists, in the plane, a sequence  $\{M_i\}$  of disjoint continua converging homeomorphically to  $M$ . Each continuum of  $C$  which does not separate the plane is atrioidic and hence is irreducible between some two points. Each continuum of  $C$  which separates the plane has only two complementary domains and is the boundary of each of them. A compact continuum belongs to  $C$  if it has uncountably many disjoint homeomorphic images in the plane. (Received October 7, 1959.)

562-9. W. G. Bade and P. C. Curtis, Jr.: Homomorphisms of commutative Banach algebras.

Continuity properties of arbitrary homomorphisms  $\nu: \mathcal{A} \rightarrow \mathcal{L}$  between commutative Banach algebras are investigated. (1) Let  $\{g_n\}$ ,  $\{h_n\} \subseteq \mathcal{A}$  such that  $\{g_n\}$  is bounded,  $h_n h_m = 0$ ,  $m \neq n$ ,  $h_n g_n = g_n$ . Then  $\sup \| \nu(g_n) \| \cdot \| h_n \|^{-1} < \infty$ . (2)  $\sup \| \nu(p) \| \cdot \| p \|^{-2} < \infty$ , the sup being over the set  $P$  of idempotents in  $\mathcal{A}$ . Thus if  $P$  is bounded so is  $\nu(P)$ . Examples show (1) and (2) are sharp. (3) Let  $\mathcal{A}$  be semi-simple, regular, with unit. There exists a finite subset  $F$  of the structure space  $\Phi_{\mathcal{A}}$  such that if  $V$  is any neighborhood of  $F$ , then  $\nu$  is continuous on  $\{f \in \mathcal{A} | f(V) = 0\}$ . (4) If  $\mathcal{A} = C(\Omega)$ , then  $\nu = \mu + \lambda$  where  $\mu$  is a continuous homomorphism coinciding with  $\nu$  on a dense subalgebra, and  $\lambda$  maps into  $\text{rad}(\mathcal{L})$ . Moreover  $\overline{\nu(C(\Omega))} = \mu(C(\Omega)) \oplus \overline{\lambda(C(\Omega))}$ . If  $\text{rad}(\mathcal{L})$  is a nil ideal  $\nu$  is continuous. If  $\nu: C(\Omega) \rightarrow \mathcal{L}$  is discontinuous there exists a homomorphism of a maximal ideal in  $C(\Omega)$  into a radical Banach algebra. No examples are known. Certain quotient algebras of  $C(\Omega)$  by nonclosed ideals are shown to be not normable. The paper extends Abstract 553-109 (Notices Amer. Math. Soc. vol. 5 (1958) . 833 ), (Received October 7, 1959.)

562-10. W. G. Bade and P. C. Curtis, Jr.: The Wedderburn principal theorem for certain commutative Banach algebras. II.

Let  $A$  be a commutative Banach algebra with unit, radical  $R$ ; and maximal ideal space  $X_A$ . By introducing a multiplication due to Arens in the second conjugate space  $A^{**}$  and using results reported in Abstract 557-35 (Notices Amer. Math. Soc. vol. 6 (1959) p. 189) it is shown that if  $A/R \simeq C(X_A)$ , and if  $R$  has finite dimension, then there exists a unique and necessarily closed subalgebra  $B$  of  $A$  such that  $A = B \oplus R$ . If the dimension of  $R$  is infinite this theorem may fail, for there exists an algebra  $A$  such that  $A/R \simeq (c)$ , the ring of convergent sequences, and such that any homomorphism of  $(c)$  into  $A$  has finite dimensional range. (Received October 7, 1959.)

562-11. Richard Block: On Lie algebras of Killing-Cartan-Seligman type.

G. B. Seligman (Memoirs Amer. Math. Soc., no. 19, 1956) proved that if  $L$  is a simple restricted Lie algebra over an algebraically closed field of characteristic  $p > 7$ , and if  $L$  has a restricted representation with nondegenerate trace form, then  $L$  is the analogue over a field of characteristic  $p$  of one of the simple Lie algebras of characteristic 0 (these algebras are called algebras of classical or Killing-Cartan-Seligman type). In this paper it is shown that the above result of Seligman may be proved without the assumption of restrictedness of the algebra (and its representation). (Received October 7, 1959.)

562-12. Ivan Niven: Sets of integers defined by approximations.

For any real  $\alpha$  and any positive  $\epsilon$ , define  $T(\alpha, \epsilon)$  as the set of positive integers  $m$  such that  $m\alpha$  is within  $\epsilon$  of an integer, that is, such that there exists an integer  $n$  satisfying  $|m\alpha - n| < \epsilon$ . Various properties of these sets are obtained, including necessary and sufficient conditions under which  $T(\alpha_1, \epsilon_1) = T(\alpha_2, \epsilon_2)$  and  $T(\alpha_1, \epsilon_1) \subset T(\alpha_2, \epsilon_2)$ . (Received October 7, 1959.)

THE NOVEMBER MEETING IN DETROIT, MICHIGAN

November 27-28, 1959

563-1. R. H. Rosen:  $E^4$  is the Cartesian product of a totally non-Euclidean space and  $E^1$ .

R. H. Bing has given an example of a decomposition space of  $E^3$  which is not a manifold at each point of a Cantor set (Ann. of Math. vol. 65 (1957) pp. 484-500), but which has the property that its Cartesian product with  $E^1$  is topologically equivalent to  $E^4$  (Bull. Amer. Math. Soc. vol. 64 (1958) pp. 82-84). Using these results an example is given of a space which is totally non-Euclidean in the sense that it contains no topological 3-cell, yet its Cartesian product with  $E^1$  is homeomorphic to  $E^4$ . From this it follows that there are involutions of  $E^4$ ,  $I^4$ , and  $S^4$  with totally non-Euclidean fixed point sets. The space is constructed by iterating a process of "cutting out" interiors of 3-cells in  $E^3$  and "pasting in" homeomorphs of Bing's decomposition space. (Received May 26, 1959.)

563-2. Philip Dwinger: Maximal field representations of a Boolean algebra.

Let  $A$  be a Boolean algebra and let  $F$  be a field (all fields are supposed to be reduced) of subsets of a set  $\mathcal{F}$ , such that  $F$  is a field representation of  $A$ .  $F$  is maximal if every field of subsets of  $\mathcal{F}$  whose elements are set-theoretic unions of elements of  $F$ , coincides with  $F$ . Let for every  $x \in \mathcal{F}$ ,  $I_x = \{X \in \mathcal{F} \mid x \in X\}$  ( $I_x$  is a prime ideal of  $A$ ).  $F$  is called open if there exists an ideal  $I$  of  $A$  such that for every prime ideal  $J$  of  $A$ ,  $J \not\subseteq I$  iff  $J = I_x$  for some  $x \in \mathcal{F}$ . (The term open is motivated by the fact that  $F$  is open iff the corresponding dense subspace of the Stone space of  $A$  is open.) Theorem 1.  $A$  is complete iff every open field representation of  $A$  is maximal. An ideal  $I$  of  $A$  is pseudo-principal if for every two ideals  $I_1$  and  $I_2$ ,  $I_1 \vee I_2 = I$ ,  $I_1 \wedge I_2 = (0)$  implies either  $I_1$  or  $I_2$  is principal. Theorem 2. With every pseudo-principal ideal of  $A$  is associated a maximal field representation of  $A$ . (Received September 25, 1959.)

563-3. J. M. Kister: Isotopies in 3-manifolds with boundaries, I.

Let  $M$  be a 3-manifold with boundary which has a given triangulation  $\Sigma$  and let  $d$  be the barycentric metric of  $M$  determined by  $\Sigma$ . Denote the set of all homeomorphisms of  $M$  onto  $M$  by  $H(M)$ , and define (for  $f, g \in H(M)$ )  $\rho(f, g) = \sup_{x \in M} d(f(x), g(x))$ .  $f$  and  $g$  are  $\epsilon$ -isotopic if there is an isotopy  $H_t$ ,  $t \in I$ , so that  $H_0 = f$ ,  $H_1 = g$  and if  $t_1, t_2 \in I$  then  $\rho(H_{t_1}, H_{t_2}) \leq \epsilon$ . Theorem. If  $\epsilon > 0$  there is a  $\delta > 0$  so that if  $\rho(f, g) < \delta$  then  $f$  and  $g$  are  $\epsilon$ -isotopic.  $\delta$  depends only on  $\epsilon$  and not on  $f$  and  $g$  or even on  $M$ . If the metric  $d$  is allowed to be arbitrary the theorem is false. For  $M$  compact this result has been obtained recently and independently by G. Fisher and M. E. Hamstrom. (Received October 1, 1959.)

563-4. D. D. Morrison: Solution of nonlinear algebraic problems.

The problem is to minimize a sum  $F$  of squares of nonlinear functions  $f$  of parameters  $a$ . To solve the problem numerically, one starts (at each iteration) with an approximation  $a_0$  and seeks a correction  $x$  to produce a better solution  $a_0 + x$ . To choose  $x$ , first the functions  $f$  are linearized; i.e., they are approximated by functions  $g$  which are linear in  $x$ . Then the sum  $G$  of squares of functions  $g$  is a quadratic function of  $x$ . At this point the usual (Gauss) method is to choose  $x$  so as to minimize  $G$ ; the difficulty is that the resulting  $x$  may be so large as to make the linearization invalid. The present method chooses  $x$  so as to minimize  $G$  under the side conditions that the components of  $x$  are less in magnitude than prescribed bounds; roughly speaking, the bounds are chosen so that the linearization is safe with such a small correction  $x$ . The choice of  $x$  is thus a quadratic programming problem which is solved (approximately), using the eigenvalues and eigenvectors of a matrix. Convergence of the method can be proved without difficulty. (Received October 2, 1959.)

563-5. Johannes de Groot: Linearization of mappings.

(1) If  $\phi: M \rightarrow M$  is a continuous map of a metrizable  $M$  into itself,  $\phi$  is topologically linear, i.e.  $M$  can be embedded topologically in some real Hilbert space  $H$ , such that the induced  $\phi$  equals a bounded linear operator  $\Phi$  of  $H$  onto itself, restricted to the embedded  $M$ .  $\Phi$  may be chosen universally, i.e. independent of  $\phi$ . (2) If  $\phi$  is a retraction,  $\Phi$  may be chosen as a projection of  $H$  into itself. (3) If  $\phi$  is an autohomeomorphism,  $\Phi$  may be chosen (universally) as a bounded linear autohomeomorphism of  $H$ . (4) Analogous results hold if a locally

compact group  $G$  of autohomeomorphisms with a countable base acts on  $M$ . If  $G$  has no countable base, the corresponding statement is false, in general. If  $G$  is compact,  $M$  can be metrized in such a way that it spans up some real Hilbert space  $H$  over which  $G$  can be extended, such that  $G$  acts as a compact group of unitary linear homeomorphisms on  $H$ . (5) If  $G$  is some set of homeomorphisms of a completely regular space, all homeomorphisms can be extended (topologically) to linear homeomorphisms of a real linear space. (Received October 5, 1959.)

563-6. O. G. Owens: A characterization in  $E^3$  of the everywhere regular solution of the reduced wave equation.

The author has already established (Trans. Amer. Math. Soc. vol. 88 (1958) pp. 388-399) a theorem which asserts that there exists on  $E^2$  a unique explicit solution of the 2-dimensional reduced wave equation for which the integral,  $F(\theta)$ , of the solution along the semi-infinite rays of a pencil of lines can be arbitrarily prescribed assuming that  $F(\theta)$  is periodic of period  $2\pi$  and satisfies a uniform Hoelder condition with exponent  $\alpha > 1/2$ . The present paper establishes the similar theorem for the 3-dimensional reduced wave equation. In the 3-dimensional case it is the integral,  $F(\theta, \phi)$ , of the solution, weighted by  $(x^2 + y^2 + z^2)^{1/2}$ , along the semi-infinite rays of a bundle of lines which is arbitrarily prescribed assuming that  $F(\theta, \phi)$  satisfies a uniform Lipschitz condition with respect to spherical distance on the surface of the unit sphere. In both the two and the three dimensional cases the integrals involved converge only conditionally. (Received October 5, 1959.)

563-7. R. L. Kelley and P. M. Swingle: Clusters of indecomposability.

Let  $Z$  and  $Z'$  be disjoint subsets of a continuum  $W$ . Let  $\{Z_i\}$  and  $\{Z'_i\}$  be classes of disjoint subsets such that  $Z = \bigcup Z_i$  and  $Z' = \bigcup Z'_i$ . Then we say that  $W$  is a continuum with cluster pair  $(Z, Z')$  of indecomposability if and only if every region-containing subcontinuum  $W'$  of  $W$  contains either  $Z_i$  or  $Z'_i$  for each  $i$  ( $i = 1, 2, \dots$ ). We show by a modification of the network construction used by R. L. Wilder for Theorem 8 [Math. Ann. vol. 109 (1933) pp. 273-306] that there exists a decomposable continuum  $W$  with cluster pair  $(Z, Z')$  of indecomposability. There exists a connected subset  $C$  of  $W$  which has the probability type property that each nondegenerate connected subset of  $C$  is dense in either  $Z_i$  or  $Z'_i$  for each  $i$ . In  $E_m$ ,  $C$  can be taken as a minimal generating set for a

topological group or algebra. (Received October 5, 1959.)

563-8. R. R. Welland: Metrisable Köthe spaces.

If  $E$  is a locally compact space and  $\mu$  a Radon measure on  $E$ ,  $\Omega(E, \mu)$  denotes the space of equivalence classes of locally integrable functions on  $E$ . Subspaces  $\Lambda$  and  $\Lambda^*$  of  $\Omega(E, \mu)$  are called associated Köthe spaces if,  $\Lambda$  resp.  $(\Lambda^*) = \{f \in \Omega: \int fgd\mu < \infty \text{ for all } g \in \Lambda^* \text{ resp. } (\Lambda)\}$ . The bilinear form  $(f, g) = \int fgd\mu$  puts  $\Lambda$  and  $\Lambda^*$  in weak duality. If  $\omega$  is the family of weakly bounded 0-convex sets in  $\Lambda^*$ , the topology of uniform convergence on sets in  $\omega$  is called the strong topology for  $\Lambda$ . A convex 0-convex set  $A \subset \Lambda$  is called admissible if every increasing sequence of nonnegative elements in  $A$  has a sup in  $A$ . Admissible sets are weakly bounded. Weakly bounded sets are contained in admissible sets. Give  $\Lambda$  and  $\Lambda^*$  their strong topologies. Theorem:  $\Lambda$  is a Banach space if and only if it contains an absorbing admissible set. Corollary:  $\Lambda$  is a Banach space if and only if  $\Lambda^*$  is a Banach space. A slight variation of these conditions tells when  $\Lambda$  is a Fréchet space. If  $\Lambda$  is a Fréchet space but not a Banach space, then  $\Lambda^*$  is not a Fréchet space. (Received October 5, 1959.)

563-9. Norman Alling: On extending valuations of an o-group to its Dedekind completion.

Let  $G$  be a totally (= simply) ordered group, which need not be Abelian, and let  $C$  be the set of all convex (= isolated) subgroups of  $G$ . Under inclusion  $C$  is a complete lattice. Let  $a \in G$  and let  $X(a)$  be the smallest convex subgroup of  $G$  containing  $a$ ; then the mapping  $a \rightarrow X(a)$  is a group valuation. Let  $Y(a) = \{x \in G: n|x| < |a| \text{ for all } n \in \mathbb{N}\}$ . Then  $Y(a) \in C$ ,  $Y(a)$  is normal in  $X(a)$  and  $X(a)/Y(a)$  is order isomorphic to a subgroup of the additive group of reals. Let  $\Sigma$  be a conditionally complete normal completion of  $G$ . It is known that addition in  $G$  can be extended to  $\Sigma$ , in general in a nonunique fashion. With such an addition in  $\Sigma$  and with an inversion in  $\Sigma$  induced by  $x \rightarrow -x$  in  $G$  we may consider the set  $C^*$  of all convex subsemigroups of  $\Sigma$  that contain 0 and are closed under inversion. A valuation again may be obtained from  $\Sigma$  into  $C^*$ . On intersecting the images of points of  $\Sigma$ , under this valuation, with  $G$ , a valuation of  $\Sigma$  into  $C$  is obtained that extends  $X$ . (Received October 5, 1959.)

563-10. G. W. Hedstrom: Equi-absolute convergence of eigenfunction expansions.

Consider the Sturm-Liouville problem:  $-y'' + ry = \lambda y$ ,  $-\infty \leq a < x < b \leq \infty$ , with linear, homogeneous boundary conditions. Here  $r$  is of bounded variation on every  $[c,d] \subset (a,b)$ . The eigenvalues are assumed to be real, discrete, and to tend to  $+\infty$ . They are also required to be "sparse enough." Assume that the eigenfunctions  $v_k$  span  $L_2(a,b)$ . Theorem. Let  $f \in L_2(a,b)$ . And for some interval  $[c,d] \subset (a,b)$  such that  $d - c < 2\pi$ , let  $f(x) = 0$  outside  $(c,d)$ . Let  $f(x) \sim \sum c_k v_k(x)$  and  $f(x) \sim \sum a_n e^{inx}$  on  $((c+d)/2 - \pi, (c+d)/2 + \pi)$ . Then  $c_k v_k(x) < \infty$  on  $(a,b)$  if and only if  $\sum |a_n| < \infty$ . As special cases we have expansions in terms of regular Sturm-Liouville eigenfunctions, Bessel functions, Jacobi polynomials, Laguerre functions, and Hermite functions. (Received October 5, 1959.)

563-11. Philip Cooperman: Notes on a nonlinear generalization of Laplace's equation.

The physical principles governing the flow of electric current in gases lead to a third order, nonlinear, partial differential equation for the electrical potential with properties analogous to Laplace's equation. In two dimensions, it can be shown that for every such potential function,  $u(x,y)$ , there corresponds a "conjugate" function,  $H(x,y)$  satisfying the "Cauchy-Riemann" equations:  $u_x = H_y/\rho$ ,  $u_y = -H_x/\rho$ , where  $\rho(x,y)$  is a continuous positive function.  $u + iH$  is a pseudo-analytic function which approaches the corresponding analytic function as  $\max \rho \rightarrow 0$ . Corresponding to the vanishing of the integral of the normal derivative of a harmonic function around a boundary, there is a theorem that  $\oint \rho u_n ds = 0$ . However, it will be shown that there are three other boundary integrals which vanish, but which have no nontrivial analogue in potential theory. (Received October 5, 1959.)

563-12. R. J. Mihalek: Modular extensions of point-modular lattices.

Let  $(\Lambda, \cup, \cap)$  be a complete complemented modular lattice satisfying (i) for  $a \neq 0$ , there exists a point  $p \leq a$ ; (ii)  $A \subset \Lambda$ ,  $p$  a point,  $p \leq \bigcup A$  implies  $p \leq \bigcup B$  for some finite  $B \subset A$ . A set  $L \subset \Lambda$  is a point-closed subsystem of  $\Lambda$  if (1)  $0 \in L$ ; (2) for nonempty  $A \subset L$ ,  $\bigcap A \in L$ ; (3) every  $a, b \in L$  possess a l.u.b. with respect to elements of  $L$ ; (4)  $a \in L$ ,  $p$  a point in  $\Lambda$  implies  $p \cup a \in L$ . Conditions on a lattice  $L$  are found that are necessary and sufficient

that  $L$  be isomorphic to a point-closed subsystem of some  $\Lambda$  (the latter is unique up to isomorphisms). These are (a) for nonempty  $A \subset L$ ,  $\bigcap A$  exists; (b) for  $a, b \in L$ ,  $a < b$ , there exists a point  $p \leq b$  such that  $p \not\leq a$ ; (c)  $L$  is point modular, i.e., for  $a, b \in L$ ,  $(p \cup a) \cap b = p \cup (a \cap b)$  for every point  $p \leq b$ . If  $\Lambda$  has at least three points on a line and length  $\geq 5$ , then  $L$  has, and conversely. Such a  $\Lambda$  can be obtained (isomorphically) as the lattice of subspaces of some linear space. These results are used in a lattice-theoretic characterization of the lattice of finite-dimensional subspaces of a linear space and the lattice of closed subspaces of a linear system in the sense of G. W. Mackey (Trans. Amer. Math. Soc. vol. 57 (1945) pp. 155-207). (Received October 6, 1959.)

563-13. J. H. B. Kemperman: Small sumsets in an abelian group.

(I) Let  $G$  be an additively written abelian group,  $A$  and  $B$  finite nonempty subsets of  $G$ ,  $o(A) + o(B) - o(A + B) = k \geq 1$ . Let  $n(r)$  denote the number of elements  $c$  having precisely  $r$  representations as  $c = a + b$ , ( $a \in A, b \in B$ ). In particular,  $n(r) = 0$  if  $0 < r < k$ , (Scherk, Amer. Math. Monthly vol. 62 (1955) p. 46). It is shown that also  $n(r) = 0$  if  $k < r < n(k)$ . If  $k = 1$ ,  $n(k) = 0$  or  $1$ , respectively, then  $A + B$  is periodic or can be made periodic by adding or deleting one element, respectively. A set  $D$  is said to be periodic if  $D + x = D$  for some  $x \neq 0$ . Similar results hold for  $k \geq 2$ . (II) Let  $G$  be an abelian locally compact group,  $\mu_*$  the inner measure induced by a fixed Haar measure on  $G$ . A simple formula is given for the exact lower bound of  $u_*(A_1 + \dots + A_n)$  subject to  $A_i \subset G$ ,  $\mu_*(A_i) = p_i$ , ( $p_i$  fixed,  $0 < p_i \leq \mu_*(G)$ ). (Received October 6, 1959.)

563-14. D. J. Eustice: Strong Riesz summability of orthogonal expansions.

A theorem of Zygmund (Fund. Math. vol. 10 (1927) pp. 356-362) and Borgen (Math. Ann. vol. 98 (1928) pp. 125-150) showing that  $(C,1)$ -summability almost everywhere of orthogonal expansions of square integrable functions implies the strong  $(C,1)$ -summability a.e. of these expansions is generalized to Riesz summability  $R(\lambda,1)$ . It is also shown that the  $R(\lambda,1)$ -summability a.e. of the partial sums of orthogonal expansions does not, in general, imply the  $R(\lambda,1)$ -summability of subsequences of the partial sums. Finally, a theorem of Tandori (Acta. Sci. Math. Szeged. vol. 20 (1959) pp. 14-18) stating a condition by which  $(C,1)$ -summability a.e. does imply strong  $(C,1)$ -summability a.e. of all subsequences of the partial sums is generalized to  $R(\lambda,1)$ -summability. (Received October 6, 1959.)

563-15. W. M. Boothby: Complex homogeneous contact manifolds.

Final report.

In Abstract 559-38 (Notices Amer. Math. Soc. vol. 6 (1959) p. 412) it was announced that corresponding to each complex, simple Lie group  $G$  there was a closed, complex subgroup  $L$ , determined up to an inner automorphism, such that the homogeneous complex manifold  $G/L$  was both Kähler and carried a homogeneous contact structure. In the preliminary report referred to these subgroups  $L$  were characterized but had not yet been enumerated. The homogeneous spaces  $G/L$  are as follows:  $SU(n+1)/SU(n-1) \times T_2$ ,  $O(2n+1)/O(2n-3) \times O(3) \times T_1$ ,  $Sp(n)/Sp(n-1) \times T_1$ , and  $O(2n)/O(2n-4) \times O(3) \times T_1$  corresponding to the Lie algebras  $A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$  respectively ( $T_1 = i$ -dim'l. torus). Corresponding to the exceptional Lie groups they are  $G_2/O(3) \times T_1$ ,  $F_4/Sp(3) \times T_1$ ,  $E_6/A_5 \times T_1$ ,  $E_7/O(12) \times T_1$  and  $E_8/E_7 \times T_1$ . (Received October 7, 1959.)

563-16. Daniel Waterman and S. A. Husain: Uniform convergence factors of orthonormal expansions.

Let  $\{\phi_n(x)\}$  be an orthonormal system over  $[0,1]$ . Each  $\phi_n$  is bounded and the system is closed in  $C$ . Necessary and sufficient conditions are found that a sequence  $\{\lambda_n\}$  be a sequence of uniform convergence factors of the expansions of functions of classes  $L^p$ ,  $p \geq 1$ , and  $B$ , the bounded measurable functions.  $\{\lambda_n\}$  is a sequence of uniform convergence factors of the functions of a certain class if for any one of these functions,  $f \sim \sum c_n \phi_n$ , we have  $\sum_0^\infty c_n \lambda_n \phi_n(x)$  uniformly convergent. Write  $\sum_0^n \lambda_\nu c_\nu \phi_\nu(x) = \int_0^1 f(t) (\sum_0^n \lambda_\nu \phi_\nu(x) \phi_\nu(t)) dt = \int_0^1 f(t) K_n(x,t) dt$ . Our conditions are: (1) for the class  $B$ ,  $\int_0^1 |K_n(x,t)| dt < M$  for all  $x$  and  $n$ , (2) for the class  $L^p$ ,  $p > 1$ ,  $\int_0^1 |K_n(x,t)|^q dt < M$ ,  $1/p + 1/q = 1$ , for all  $x$  and  $n > N$ , (3) for the class  $L$ ,  $\text{ess. sup.}_{0 \leq t \leq 1} |K_n(x,t)| < M$  for all  $x$  and  $n$ . These results extend results of Karamata [J. Math. Pures Appl. vol. 35 (1936)] on trigonometric series and Aljancic [Publ. Inst. Math. Serbe Sci. vol. 10 (1956)] on orthonormal expansions of functions of class  $C$ . (Received October 7, 1959.)

563-17. J. P. Roth: Algebraic topological methods for the synthesis of switching systems. IV. Minimization of singular Boolean trees.

Let  $f$  be a Boolean function of  $n$  variables. Let  $F$  be a functional expression of  $f$ , where the primitive functions belong to a specified class  $B$ . Let a

positive integer be attached to each primitive of  $B$ , termed its cost. Let the cost of a functional expression be the sum of the costs of the elements used in  $F$

Problem: To devise an algorithm such that, given any  $f$ , the algorithm compute a functional expression for  $f$ , of minimum cost. This paper gives such an algorithm. A special case is the well known classical problem of finding a minimal functional expression where  $B$  consists of the logical elements AND, OR and NOT. The point of view is combinatorial topological. (Received October 7, 1959.)

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