



Division of
Academic Enhancement
UNIVERSITY OF GEORGIA



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dae.uga.edu/tutoring/math-statistics

math.uga.edu/studyhall

MATH 1101 Chapter 3 Review

Topics Covered

Section 3.1 Fundamental Concepts of Linear Functions

Section 3.2 Modeling with Linear Functions

Section 3.3 Linear Functions and Data

Section 3.4 Linear Regression: Finding the Line of Best Fit

How to get the most out of this review:

1. Watch the video and fill in the packet for the selected section. (Video links can be found at the two web addresses at the top of this page)
2. After each section there are some 'Practice on your own' problems. Try and complete them immediately after watching the video.
3. Check your answers with the key on the last page of the packet.
4. Go to office hours or an on-campus tutoring center to clear up any 'muddy points'.

Section 3.1 Fundamental Concepts of Linear Functions

Linear Function

A function is linear if it can be written in the form $y = ax + b$ where a is the slope of the line and b is y -intercept. Functions where the ARC is constant are linear functions where the ARC is the slope.

Intercepts

The y -intercept (or vertical intercept) is where the line crosses the y -axis and is located at $(0, b)$. The x -intercept (or horizontal intercept) is where the line crosses the x -axis and is located at $(x, 0)$.

Example 1

Find the intercepts of the linear function $f(x) = 6x + 36$.

Example 2

Julie had \$1025 in savings on June 1, 2010 when she graduated from college. Her new job will pay her \$2200 per month after taxes and monthly expenses. Answer the following

- (a) Find a linear function that gives Julie's net income

- (b) How much money will she have made by the end of October? (Assume she is paid on the 15th of each month)

- (c) How long until Julie has made \$25,000?

- (d) In January, Julie's student loan payments begin. If she has a balance of \$75,000 and her payments are \$500 per month, find a linear function for the balance due on her loans. (Assume Julie got an interest free loan from a rich relative)

- (e) How long until Julie's loan balance falls below \$50,000?

Practice on Your Own

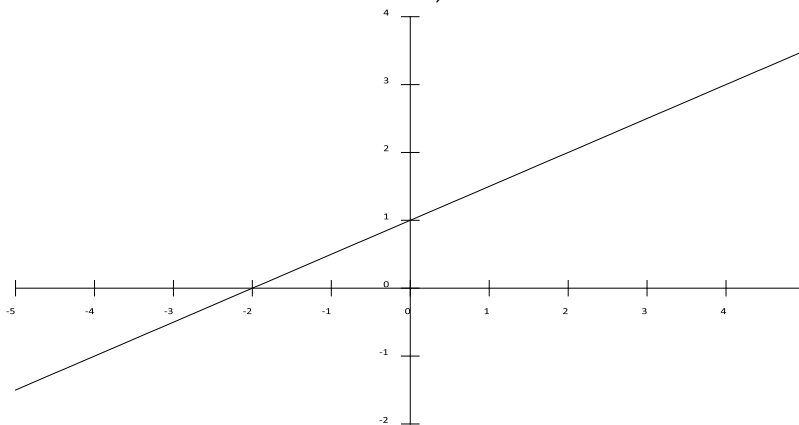
1. Your lemonade stand on North Campus sold 59 cups when your price was \$0.50 per cup. The next day you changed the price to \$0.75 and sold 44 cups.
- (a) Assume the function between sales and price is linear. Write a function $s(p)$ where s is the number of cups sold and p is the price charged.

(b) How many cups do you sell if you charge \$1.00?

(c) You only brought 20 cups, you want to set your price so you sell all of them. What should you charge your customers?

(d) How much money do you make selling those 20 cups for the price you found?

2. Is this a linear function of x ? If so, what is the function?



Section 3.2 Modeling with Linear Functions

Working with two linear models

If two linear models are using the same input and output variables, we can find where the models are equal by looking at their intersection.

Example 3

Two different student groups at UGA are fundraising for Relay For Life. Group A's fundraiser is modeled by $f(t) = 255t + 322$ and group B's fundraiser is modeled by $g(t) = 145t + 536$ where t is measured in weeks. If the two groups start their fundraiser at the same time, how long until they have raised the same amount?

Example 4

Bill just got a new sales job. He can take a base salary of \$10,000 per year plus 20% of his sales revenue or he can take a base salary of \$50,00 plus a 5% of his sales revenue.

(a) Find the linear models for his annual salary for each of the two options.

(b) How much will Bill earn under each plan if he sells \$40,000 worth of product?

(c) How much should Bill sell to make more money off the first option? How much would he make that year?

Example 5

Makebelievia, Inc. is making automatic homework machines. They can make 2000 the first year, increasing by 250 every year. 2750 people want to buy them the first year, increasing by 100 every year. What's the first year that everyone who wants one will be able to buy one? (Hint: number made \geq number wanted)

Piecewise Functions

Functions whose ARC only stays constant for different intervals of the input value. We “piece” portions of different linear functions together.

Example 6

Christy, Tim and Dory are taking a road trip to see their favorite band Family and Friends. They leave Athens at noon and drive for 4 hours at 60mph, they stop for lunch for an hour, get back on the road and drive 50mph for 2 more hours and when they hit town there is a traffic jam so only go 20mph for the remaining half hour.

(a) Sketch a function of distance vs. time for their journey.

(b) What is the formula for the distance function?

(c) What time do they arrive?

(d) How far was their total drive?

(e) How far had they traveled by 5:30PM?

Practice on Your Own

1. You are heading to the Georgia vs. Florida game! You begin your trip traveling 60 mph for 3 hours. You then decide to stop for lunch for an hour. After lunch, you continue your trip driving 55 mph for 2 hours. Once you enter Jacksonville you hit heavy traffic before you reach the stadium. During traffic you travel 20 mph for half an hour until you are able to park. Go DAWGS! Let $d(t)$ represent the total number of miles that you have traveled and let t represent the number of hours.

(a) Write a formula for your distance traveled in terms of number of hours.

(b) What is the domain and range of your function?

(c) How far have you traveled after five and half hours?

(d) When are you exactly 100 miles from home?

2. Georgia Power charges different rates depending on how much power a customer uses. The amount charged is modeled by $f(x)$ where x is the number of kWh used.

$$f(x) = \begin{cases} 15 + 0.15x, & 0 \leq x \leq 100 \\ 5 + 0.25x, & 100 < x \leq 2000 \\ 305 + 0.10x, & 2000 < x \end{cases}$$

(a) What is your power bill if you use 80 kWh?

(b) What is your power bill if your roommate leaves their space heater on high over the break and you use 280 kWh?

(c) What is your power bill if you are running an industrial aluminum refinery and you use 6000 kWh?

(d) If your power bill is \$103, how much electricity did you use?

(e) What is the average rate of change over the interval $[10,50]$?

(f) Describe in words, what does $f(x) = 15 + 0.15x$ mean?

Section 3.3 Linear Functions and Data

How do we determine if a function is linear from looking at a table of data points?

You could plot the data points but looks can be deceiving. You need to verify that the function is truly linear! A linear function will have a constant ARC.

Example 7

Are the two following functions linear? If yes, give the linear model.

x	5	10	15	20	25
y	2.46	3.81	6.43	7.78	9.19

x	5	10	15	20	25
y	5.53	8.04	10.55	13.06	15.57

Practice on Your Own

1. Is this a linear function of x ? If yes, what is the function?

x	0	1	2	3	4	5
y	3.500	.350	-.954	-1.956	-2.80	-3.54

2. Is this a linear function of x ? If yes, what is the function?

x	0	1	2	3	4	5
y	7.100	3.950	.800	-2.350	-5.500	-8.650

Section 3.4 Linear Regression: Finding the Line of Best Fit

How do we work with functions or data that are 'almost' linear?

If the data points show that the function behaves linearly but doesn't have a constant ARC, we can use Linear Regression to find the line of best fit that will approximate a model for us.

Linear Regression

A computational technique that produces a linear model that best fits the data set. The line of best fit is determined by minimizing the SSE (sum of squared errors) and AE (average error)

Model Error

Since the actual data points may or may not lie on the line of best fit, the model error for a specific data point is calculated by finding the vertical distance between the point and the line.

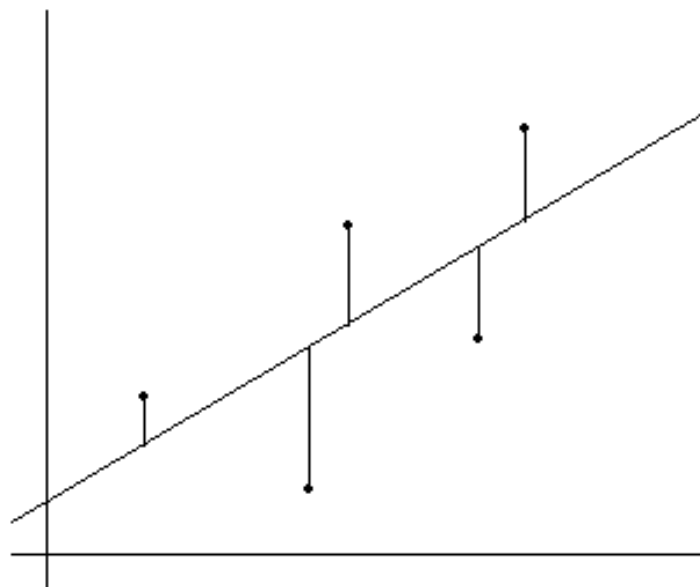
$$\text{Error} = \text{actual value} - \text{predicted value}$$

The actual value is the y value of the data point for a given x and the predicted value is the y value the model gives for the same value x .

- Data points that lie above the line will produce positive errors and the model under predicts
- Data points that lie below the line will produce negative errors and the model over predicts
- The average error (AE) of the model is found by using the SSE (Sum of Squared Errors) where n is the total number of data points in the set

$$AE = \sqrt{\frac{SSE}{n}}$$

What does it look like?



Finding the Line of Best Fit in the Calculator

1. Go to STAT→Edit...
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Go to STAT→CALC→LinReg(ax+b)
5. Type L_1, L_2, Y_1 (Y_1 is under VARS)
6. You will see the values for a, b and the line will be stored in Y_1

Example 8

Find the line of best fit for data set 1.

Data Set 1							
x	63	64	66	69	71	73	75
y	127	121	142	162	156	181	208

Finding the SSE in the Calculator

1. Go to STAT→Edit...
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Enter $Y_1(L_1)$ in L_3
5. Enter $L_2 - L_3$ in L_4
6. Enter L_4^2 in L_5
7. Go to 2nd →STAT→MATH→sum(L_5)

Example 9

Find the SSE and AE for data set 1. Round your answers to 2 decimal places.

SSE:

AE:

Percent Error

We can calculate the % error for a specific data point. $\frac{(\text{actual}-\text{predicted})}{\text{actual}} \times 100\% = \% \text{ error}$

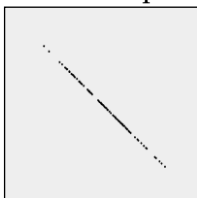
Example 10

Find the percent error for $x = 63$ and $x = 75$ for data set 1.

Correlation Coefficient, r

The correlation coefficient measures the *strength* of the relationship of the two variables used in the model. r can be any value in the interval $[-1,1]$. The sign of r matches the sign of the slope.

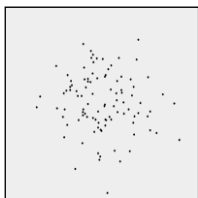
Below is a visual representation of r



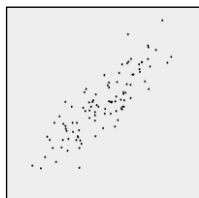
$r = -1$



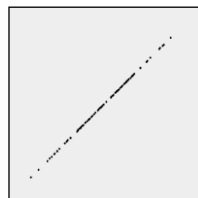
$r = -.50$



$r = 0$



$r = +.85$



$r = +1$

You can see the r value for any model you build using LinReg by turning the Diagnostics On.

Example 11

What is the correlation coefficient for the model for data set? What can we say about the two variables in the model?

Example 12

In data set 1, if x is the number of people in a room and y is the total of one dollar bills in their pockets, how many people should be in the room to have \$150 in one dollar bills?

Practice on Your Own

1. The following data represents the number of points scored in the Makebelievia national championship mudslinging match for several years.

Year	2003	2004	2005	2006	2007	2008
Points	103	131	143	172	184	219

- (a) Find the linear model $M(t) = at + b$ that best fits these data where t is the number of years after 2003.
- (b) Find the SSE of your model.
- (c) Find the AE (average error).
- (d) Find the correlation coefficient.
- (e) Can you be 95% confident that there is a correlation between year and points awarded if $r = 0.878$?
- (f) The model [*underestimates/overestimates/exactly predicts*] the points in 2006.
- (g) Use the model to predict the number of points in 2011.

Answers to the Practice on Your Own problems

Section 3.1

- (a) $s(p) = -60p + 89$
(b) 29
(c) \$1.15
(d) \$23
- Yes. $y = \frac{1}{2}x + 1$

Section 3.2

- (a)

$$d(t) = \begin{cases} 60t + 0, & 0 \leq t \leq 3 \\ 0t + 180, & 3 < t \leq 4 \\ 55t - 40, & 4 < t \leq 6 \\ 20t + 170, & 6 < t \leq 6.5 \end{cases}$$

- (b) Domain: $[0, 6.5]$, Range: $[0, 720]$
(c) $d(5.5) = 262.5$
(d) $t = 1.67$ hours, 1 hr 40 min
- (a) \$27
(b) \$75
(c) \$905
(d) \$392 kWh
(e) 0.15
(f) There is a 15 dollar monthly charge and every kWh costs \$0.15,

Section 3.3

- Answer
- Answer

Section 3.4

- (a) $M(t) = 21.94t + 103.81$
(b) 143.28
(c) 4.89
(d) 0.9916
(e) Yes
(f) The model **underestimates** the points in 2006.
(g) 279.35