By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	20	
2	10	
3	15	
4	20	
5	15	
6	22	
7	10	
8	5	
9	10	
10	10	
11	10	
12	10	
13	20	
14	10	
Total:	187	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. Please determine all values of x that satisfy each equation below. Print your answer(s) in the box provided.

_____ (a) [5 pts] $\dfrac{1}{3-x} = \dfrac{5}{x+1}$

$x =$

_____ (b) [5 pts] $\ln(x+1) + \ln(x) = 3$

$x =$

(c) [5 pts] $6 \cdot 5^x = 12^x$

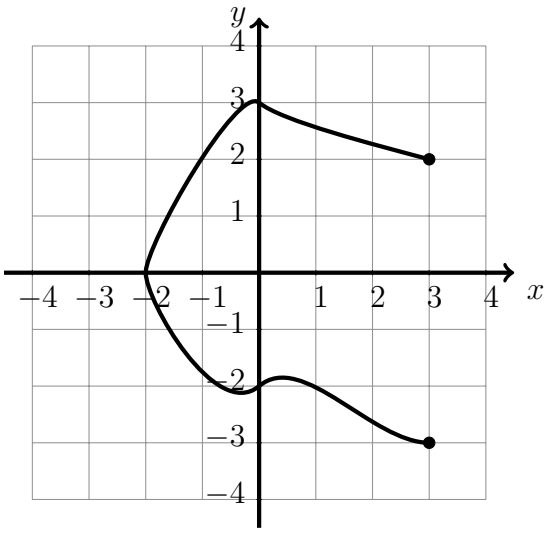
$x =$

(d) [5 pts] $\tan(x) = 1.4$ where x is in the third quadrant and $0 \leq x < 2\pi$. (Your answer should be in radians and can be the exact answer or approximate the answer to within two decimal places.)

$x =$

2. Please determine the x and y intercepts for each relationship given below. Also determine if each relationship given below is a function, and briefly explain your reasoning.

_____ (a) [5 pts] The graph of the relationship is given in the figure below. (Determine the intercepts and whether or not y is a function of x.)



_____ (b) [5 pts] The relationship given by

$$y + x^2 + 1 = 0.$$

(Determine the intercepts and whether or not y is a function of x.)

3. A function is given by

$$h(x) \;=\; 3 \cdot e^{-5x}.$$

_____ (a) [6 pts] Please determine the domain and range of the function.

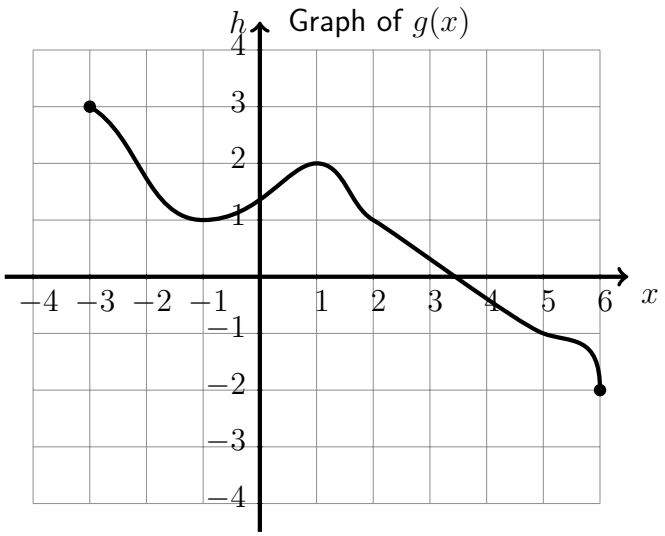
(b) [6 pts] Please determine the average rate of change of the function from $x = 1$ to $x = 3$. (Your answer should be exact.)

_____

(c) [3 pts] Please determine the distance between the two points on the graph of the function from $x = 1$ to $x = 3$. (Your answer should be exact.)

_____

4. The graph of a function, g, is given in the plot below.



(a) [5 pts] Please determine the domain and range of the function.

(b) [5 pts] Please determine the intervals where the function is increasing and where it is decreasing.

(c) [5 pts] Please determine any local minimums and local maximums. (Do not include the endpoints.)

(d) [5 pts] Does an inverse exist for this function? (Briefly explain how you reached your conclusion.)

5. For each scenario below circle the phrase that best describes the **kind** of function that will best approximate the phenomena described.

_____ (a) [5 pts] The number of ovens assembled given the time of day for a factory in which a constant 150 ovens are produced per hour.

Linear Function	Quadratic Function	Exponential Function	Logistic Function	Trigonometric Function

_____ (b) [5 pts] The number of hours of sunlight in a day given the day of the year.

Linear Function	Quadratic Function	Exponential Function	Logistic Function	Trigonometric Function

_____ (c) [5 pts] Amount of money in a bank account with compounded interest and no withdrawals.

Linear Function	Quadratic Function	Exponential Function	Logistic Function	Trigonometric Function

6. Two functions, u and w are given below. Use the definitions of the functions to answer the questions below. Show all of your work or explain how you determined your result. If a value does not exist briefly explain why not.

$$u(x) = x^2.$$

x	$w(x)$
0	2
1	0
2	1
3	-1
4	-2

_____ (a) [6 pts] Please determine the exact value of $u(w(3))$

(b) [6 pts] Please determine the exact value of $w^{-1}(2)$

_____

(c) [6 pts] Please determine the exact value of $w^{-1}(u(0))$

(d) [4 pts] Please determine the exact value of $u^{-1}(w(0))$

_____

7. Express each of the following polynomials in factored form.

(a) [5 pts] $2x^2 + 5x - 3$

(b) [5 pts] $x^3 - 2x^2 - 40x - 64$ (Note: One of the roots of the polynomial is $x = -4$.)

8. [5 pts] Please determine a quadratic function that has its vertex at $(3, -2)$ and has a y-intercept at $y = 6$. Is this the only quadratic function that satisfies the given requirements?

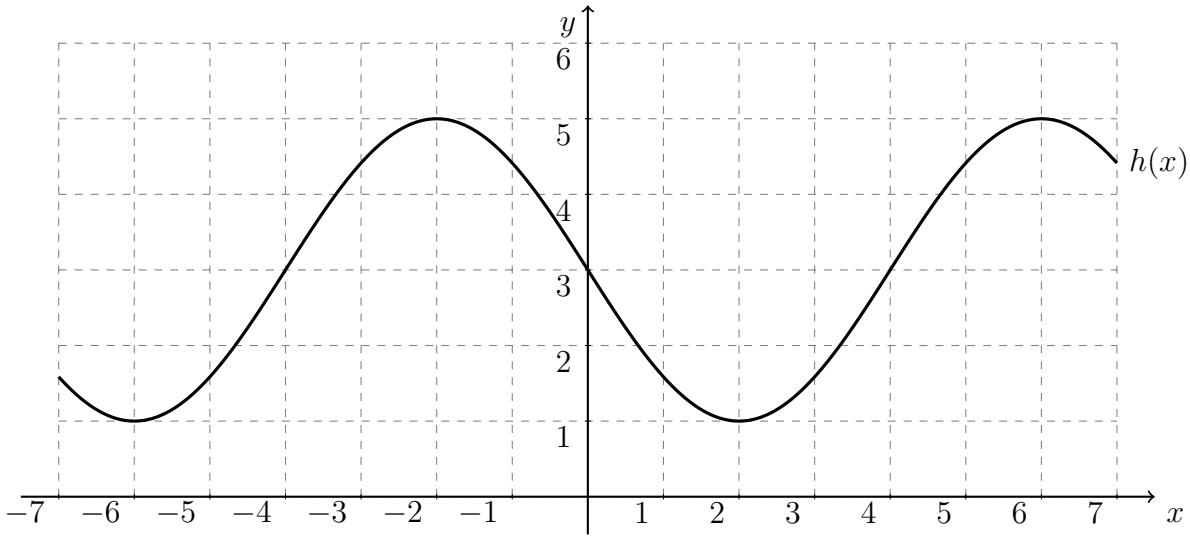
9. [10 pts] The Dread Pirate Bobby is watching a Spanish galleon from the crow's nest, and he is 20m above the surface of the water. The angle of depression for the pirate's line of site is 3°. How far is the Spanish galleon from the pirate ship? (Your answer can be the exact answer or approximate the answer to within two decimal places.)

10. A bank offers a new account that guarantees customers will receive a fixed interest that will be compounded monthly.

    (a) [5 pts] A customer initially invests \$1,000. If the interest is 3.5% compounded monthly how long must a customer keep their money in the account so that their initial investment will increase by a total of 25%?

    (b) [5 pts] The bank wishes to guarantee that a customer's initial investment will increase by 15% in five years. What should the monthly compounded interest rate be?

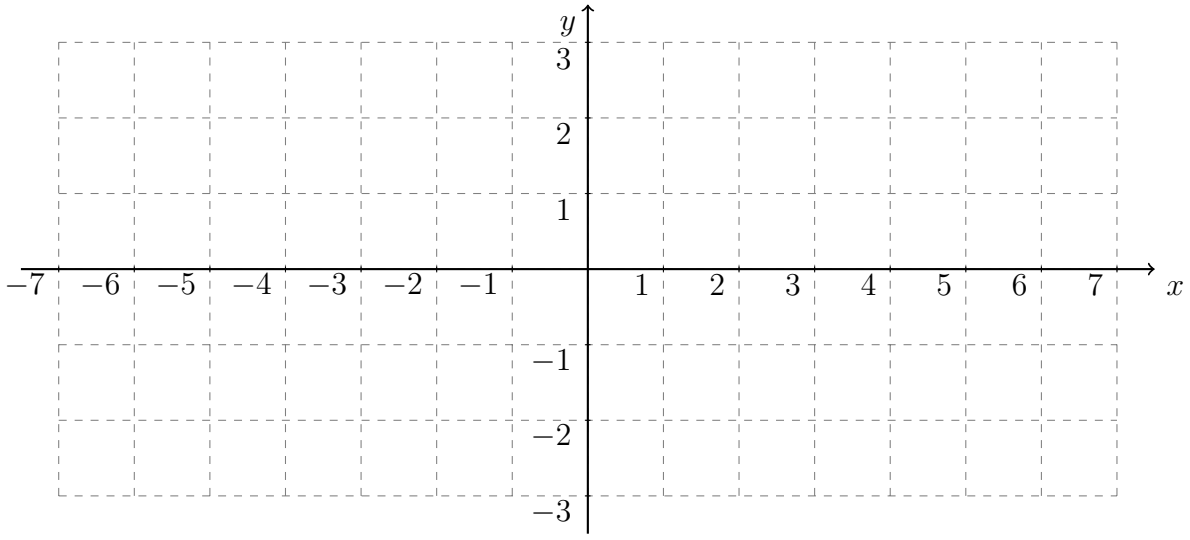
11. (a) [5 pts] Please determine a formula for the function below expressed as a sine function.



(b) [5 pts] Use the axis below to make a rough sketch of the function

$$h(x) \;=\; \cos\left(\frac{\pi}{3}x - \frac{\pi}{3}\right) - 2.$$

Indicate and label two local maxima and one local minima.



12. The body volume versus the brain volume for an order of beetles, Coleoptera, was examined in a paper by Polilov and Makarova.[1] The authors found that the volume of an insect's brain was related to the volume of the insect's body by the relationship

$$\text{Brain Volume} \;=\; 0.0800 \cdot (\text{Total Volume})^{0.699}\,.$$

(All volumes are measured in nanoliters (nl).)

_____ (a) [5 pts] A beetle is captured, and its total volume is estimated to be 250nl. What is the estimated volume of its brain? (Your answer can be exact or should be to within 0.01nl.)

_____ (b) [5 pts] A fragment of an ancient beetle is found in a piece of amber, and the volume of its brain is estimated to be 50nl. Assuming that the relationship above still holds, determine an estimate for the volume of the whole beetle. (Your answer can be exact or should be to within 0.01nl.)

[1] The scaling and allometry of organ size associated with miniaturization in insects: A case study for Coleoptera and Hymenoptera, Polilov, Alexey A. and Makarova, Anastasia A., *Scientific Reports*, Number 7, 22 Feb 2017.

13. [10 pts] You have two decaying substances, Blue Stuff and Red Stuff. The Blue stuff initially has a mass of 10 mg, and one month later it goes down to 7.7 mg, and one month after that it goes down to 5.4 mg. The Red Stuff initially has a mass of 9 mg, then one month later goes down to 6 mg, then one month after that it goes down to 4 mg.

(a) [5 pts] One quantity is decaying linearly, the other one is decaying exponentially. Which is which? (Briefly explain your reasoning.)

(b) [5 pts] Please determine the equation for the amount of the substance that is decaying exponentially as a function of time.

14. [10 pts] A game for small children will be constructed, and it includes two squares. The sum of the perimeters of the two squares will be ten centimeters. Determine the dimensions of the two squares that will minimize the total combined area. (Demonstrate that your answer is correct and do not make a guess based on intuition.)

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____