By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Total Points Possible	Points Made
1	0	
2	30	
3	20	
4	15	
5	20	
6	15	
7	15	
8	10	
9	15	
10	15	
11	20	
12	10	
13	10	
14	15	
Total:	210	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. This will make it easier to properly grade and give you the credit you deserve.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, and answers should be given in a form that can be directly entered into a calculator. If you give a numerical value it should be to within one decimal place unless otherwise stated.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: On a scale from 1 to 10, how badly was Shirin van Anrooij treated by her teammates?

2. Determine all values of x that satisfy each equation below.

   (a) [5 pts] $x^2 = 3x + 8$.

   (b) [5 pts] $3 = \sqrt{1 - 5x}$.

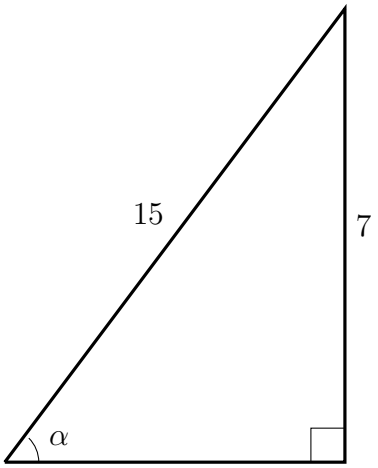
(c) [7 pts] $\ln(x-3) - \ln(x+2) = -5$.

(d) [5 pts] $14 - 3^{7-6x} = 10$.

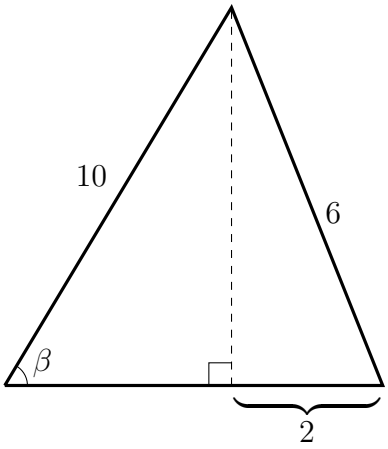
(e) [8 pts] $13 \cdot 5^{1+x} = 4 \cdot 6^{3-2x}$.

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine its numerical value**.)

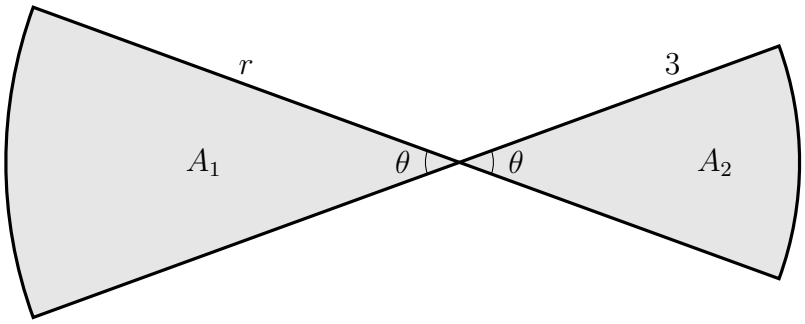
   (a) [5 pts] Determine the sine, cosine, and tangent of the angle α as shown in the diagram below:



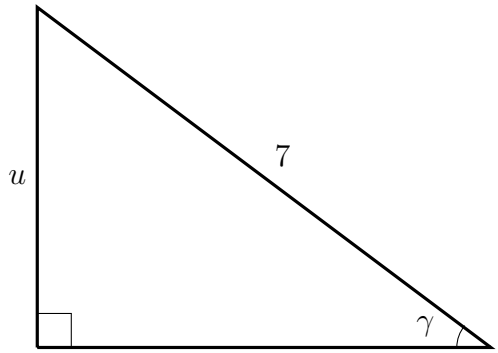
   (b) [5 pts] Determine the value of $\sin(\beta)$ in the diagram below.



(c) [5 pts] The area in the left sector, A_1, is twice the area of the right sector, A_2. Determine the value of r. (The two sectors have the same central angle.)



(d) [5 pts] Determine the value of u so that $\cos(\gamma) = 0.6$. The value of u should be a positive number.



4. The questions below refer to the two functions

$$\begin{aligned} h(x) &= 3e^{-2x}, \\ p(x) &= 2e^{-x}. \end{aligned}$$

(a) [5 pts] Which function, $h(x)$ or $p(x)$, decreases faster? (Provide a brief justification of your conclusion.)

(b) [5 pts] For what values of x is $h(x)$ greater than $p(x)$? (Your answer should be in interval notation.)

(c) [5 pts] A new function is defined, $k(x) = p(ax)$, where a is a constant. For what values of a is the new function, $k(x)$, an increasing function? (Your answer should be in interval notation.)

5. Determine the **exact** numerical values of each expression below. (Do not use a calculator.) Your final answer should not have any trigonometric, logarithmic, or exponential functions. (Your answer can be left as a product/sum of constant values. You do not have to evaluate the final expression.)

   (a) [5 pts] $\cos(\arcsin(2x))$. (Your final answer should be a function of x.)

   (b) [5 pts] $\sin(\arccos(0.2) - \arctan(-3.4))$.

(c) [5 pts] $\log_b\left(\frac{u^3}{v^2}\right)$ where $\log_b(u) = 1.8$, and $\log_b(v) = -3.2$.

(d) [5 pts] $\arcsin\left(\sin\left(\frac{6\pi}{5}\right)\right)$.

6. In each question below a line is described. Determine a **formula** (or relationship) for the line based on the description, **and** state whether or not the line is increasing or decreasing.

   (a) [5 pts] The line through the points $(2, 3)$ and $(-1, 4)$.

   (b) [5 pts] The line whose x-intercept is $(3, 0)$, and the line is parallel to the line $7x + 3y + 6 = 0$.

   (c) [5 pts] The line whose graph includes the point $(4, 7)$, and when x increases by 6 the value of y increases by 5.

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] Starting at noon, an object is placed in an oven, and the temperature of the object increases by 0.7° Celsius every 3 minutes. *The temperature of the object as a function of the number of minutes since noon.*

   Linear Function | Quadratic Function | Exponential Function | Trigonometric Function

   (b) [5 pts] Starting at noon, an object is placed in a cooler, and the temperature of the object decreases by 3% of its current temperature every five minutes. *The temperature of the object as a function of the number of minutes since noon.*

   Linear Function | Quadratic Function | Exponential Function | Trigonometric Function

   (c) [5 pts] Starting at sunrise, the temperature of an object increases until sunset, and then its temperature decreases. The cycle continues the next day. *The temperature of the object as a function of the number of minutes since sunrise on the first day.*

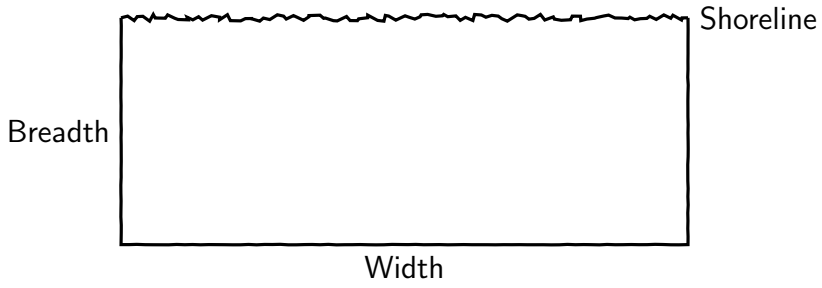
   Linear Function | Quadratic Function | Exponential Function | Trigonometric Function

8. [10 pts] Verify the identity

$$\frac{1-\tan(\alpha)\tan(\beta)}{\tan(\alpha)-\tan(\beta)} = \frac{\cos(\alpha+\beta)}{\sin(\alpha-\beta)}.$$

9. [15 pts] A park is in the shape of a rectangle, and the width of the rectangle is 500m. The border at the top of the rectangle is a river, and the breadth of the rectangle shrinks due to erosion. The breadth decreases by 5% each year. Initially, the breadth is 100m.

   Determine the area of the park as a function of the time in years. How long will it take until the area is reduced by half of its initial area?



10. [15 pts] A toy car is moving on an inclined track, and the force due to gravity pulls the car down while friction also acts on the car. The car starts from rest (zero velocity), and after five seconds its velocity is estimated to be 0.25 m/sec. The velocity of the car in meters per second is approximated by

$$v(t) = 0.35 - Ae^{rt},$$

where t is the time in seconds. Determine how long it will take until the car is moving at a velocity of 0.3 meters per second.

11. Each of the following questions refer to the functions

$$\begin{aligned} q(x) &= -2(x-7)^2+4, \\ b(x) &= -2(x-a)^2+4, \end{aligned}$$

where a is a constant.

(a) [3 pts] Determine the y-intercept of $q(x)$.

(b) [5 pts] Determine the x-intercepts of $q(x)$.

(c) [5 pts] Determine the set of values of x where $q(x)$ is increasing. (Your answer should be stated using interval notation and briefly justify your conclusion.)

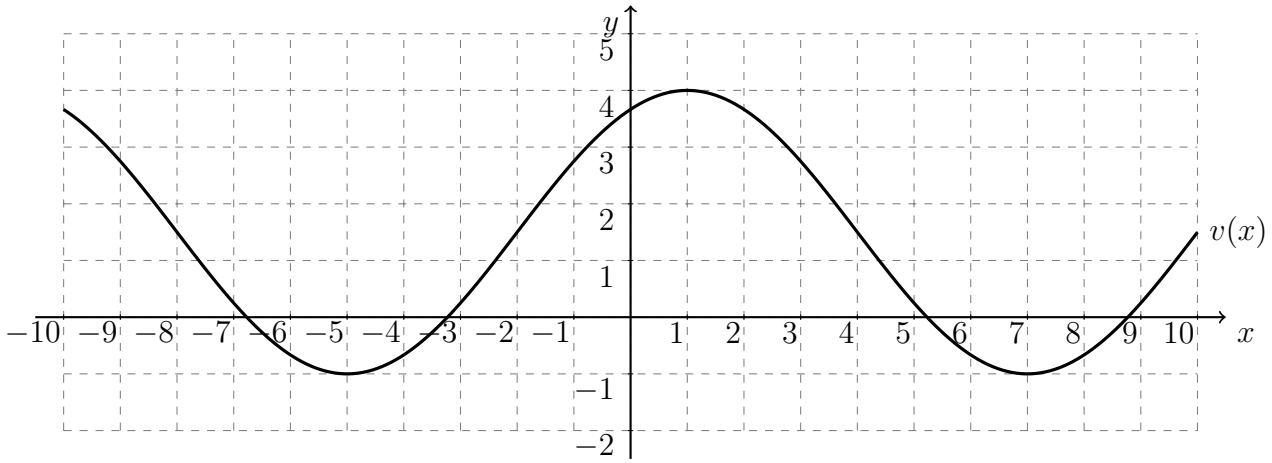
(d) [7 pts] Determine the values of a for which all of the x values of the x-intercepts of $b(x)$ are positive. (Your answer should be stated using interval notation.)

12. [10 pts] Paul Atreides is standing on a mountain top looking down at the city of Arrakeen. Paul spots an armoury at an angle of depression of 12°. Paul knows that the distance between the armoury and the base of the mountain is 3,500 meters. How tall is the mountain top above the city?

13. [10 pts] Determine a formula for the function whose graph is shown below expressed as a sine function,

$$v(x) \;=\; A \cdot \sin(bx + c) + d.$$

The values of A and b should be positive numbers.



$A =$

$b =$

$c =$

$d =$

14. [15 pts] A bird sanctuary consists of two types of area: an area of x square meters with land area, and an area of y square meters with a water surface. The total of these two areas must cover the whole 300,000 square meters of the sanctuary. The sanctuary hosts a species of duck, whose survivability depends on these area amounts.

    Land area survivablity index The land survivability index is a linear function of the amount of land area x. This index is 0.0 when there is no land area, and it increases by 3 units for each additional square meter of land area.

    Water area survivablity index The water survivability index is a linear function of the amount of water area y. This index is 2.0 when there is no water area, and it increases by 4 units for each additional square meter of water area.

    Determine the amounts of land area and water area that maximize the **product** (*not sum*) of the survivability indices.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____