By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): ______________________ Name (print): ______________________

Student Number: ______________________

Instructor's Name: ______________________ Class Time: __________________

Problem Number	Total Points Possible	Points Made
1	0	
2	25	
3	20	
4	20	
5	10	
6	18	
7	15	
8	10	
9	15	
10	15	
11	10	
12	10	
13	10	
14	10	
Total:	188	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, and answers should be given in a form that can be directly entered into a calculator. If you give a numerical value it should be to within one decimal place unless otherwise stated.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: Was it right to disqualify Marlen Reusser for riding her bike on a bike path?

2. Determine all values of x that satisfy each equation below.

(a) [5 pts] $\dfrac{1}{x+3} = \dfrac{5}{2x-1}$

(b) [5 pts] $\sqrt{x^2 - x} = 2x + 1$

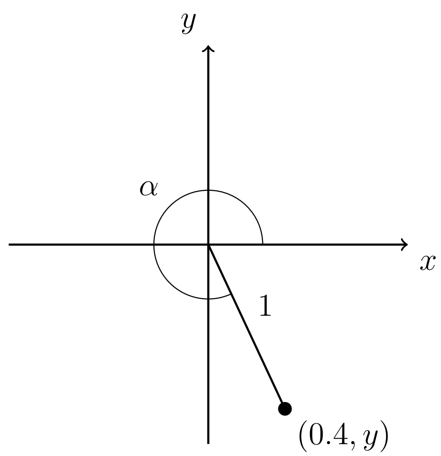
(c) [5 pts] $\dfrac{e^{3x}+1}{5}=7$

(d) [5 pts] $\log_2(7-3x)=2+\log_2(x+7)$

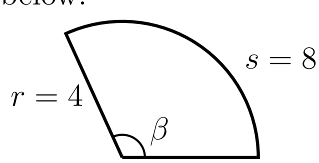
(e) [5 pts] $6 \cdot 5^x = 8 \cdot 12^{9-4x}$

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine their numerical value**.)

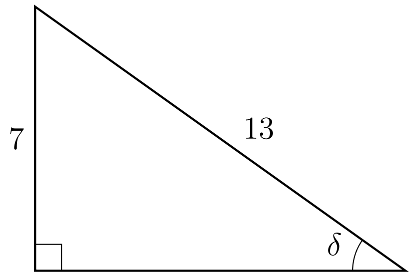
   (a) [5 pts] Determine the cosine, sine, and tangent of the angle α in the diagram below. The coordinate, $(0.4, y)$, is a point on the unit circle.



   (b) [5 pts] Determine the radian measure of the angle β in the sector shown in the diagram below:



(c) [5 pts] Determine the sine, cosine, and tangent of the angle δ in the diagram below.



(d) [5 pts] Determine the **exact** numerical value of the following expression and express it without the use of any trigonometric functions:

$$\cos\left(\arctan(1.7) + \arcsin(0.01)\right).$$

(Show your work and do not provide a numerical estimate or use a calculator.)

4. Answer each of the following questions using the functions defined below:

   $g(x) = \sqrt{x}$, $\qquad h(x) = \ln(2x - 1)$

   (a) [5 pts] Determine the inverse of $h(x)$.

   (b) [5 pts] Determine the value of $g\left(h\left(1\right)\right)$. (Show your work and each step. Do not simply write down the final number.)

   (c) [5 pts] Determine the domain of $h(x)$. (Your answer should be an interval. Use interval notation.)

   (d) [5 pts] Determine the domain of $h(g(x))$. (Your answer should be an interval. Use interval notation.)

5. Two functions are defined to be

$$\begin{aligned} f(x) &= 3e^{2x}, \\ g(x) &= e^{ax}, \end{aligned}$$

where a is a constant.

(a) [5 pts] What values of a will guarantee that the function $g(x)$ will be an increasing function. (Your answer should be written as an interval.)

(b) [5 pts] What values of a will guarantee that the graphs intersect at $x = 2$?

6. The function $l(x)$ is a linear function whose graph includes the points $(3, 4)$ and $(-1, 8)$. The function $q(x)$ is a quadratic function whose maximum on its graph is located at the point $(2, 5)$, and the y-intercept of the graph is located at $(0, -7)$.

   (a) [5 pts] Determine the formula for $l(x)$.

   (b) [5 pts] Determine the formula for $q(x)$.

   (c) [5 pts] Determine a line parallel to $l(x)$ that includes the coordinate $(1, -1)$.

   (d) [3 pts] Suppose that the function $q(x)$ had a different y-intercept. What y-intercept would result in a function that is not a quadratic?

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The total cost of risk for a project increases by 1.4% each year. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

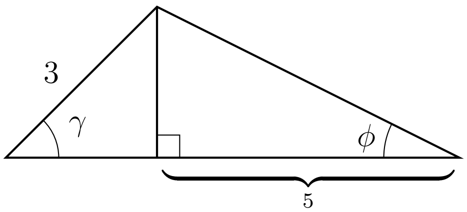
   (b) [5 pts] The total cost of risk for a project increases during warmer months and decreases during colder months. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (c) [5 pts] The total cost of risk for a project increases by $8,000 each year. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

8. [10 pts] Determine the radian measure of the angle ϕ in the diagram below. The radian measure of the angle γ is $\frac{\pi}{4}$. (If you use a calculator to provide an approximation, the approximation should be within two decimal places.)



9. Determine the formulas for the functions described in each statement below.

   (a) [5 pts] An institution offers a savings account that has an 1.3% annual interest compounded monthly. An account is opened with an initial balance of \$3,000. Determine the function that gives balance of the account given the number of years after it is opened.

   (b) [5 pts] An exponential function, $p(x) = Ce^{rx}$, whose graph includes the points $(0, 4)$ and $(5, 8)$.

   (c) [5 pts] The function whose inverse is $g^{-1}(x) = \dfrac{1}{1+x}$.

10. A sled slides down a hill and its speed increases. When it reaches the bottom, the path levels out, and the speed decreases. The speed of the sled is approximated by the following function of the time (seconds),

$$v(t) = \begin{cases} 1.7t & 0 \leq t \leq 5, \\ 8.5e^{-0.3(t-5)} & t > 5. \end{cases}$$

(a) [5 pts] Is the function a one-to-one function? (Briefly explain your reasoning. You can use algebraic, graphical, or an informal written argument to justify your conclusion.)

(b) [5 pts] What is the maximum velocity?

(c) [5 pts] Determine the average rate of change in the speed from the initial time ($t = 0$ seconds) and $t = 10$ seconds. (If you use a calculator to provide an approximation, the approximation should be within two decimal places.)

11. When a drug is administered to a patient there is a probability that the patient will experience an adverse reaction. It is assumed that the probability depends on the dosage (in milligrams) given to the patient. One model that is used to approximate the probability, p, given the dosage, x mg, is

$$m \cdot x + b \;=\; \ln\left(\frac{p}{1-p}\right),$$

where m and b are constants[1].

(a) [5 pts] In experiments it is estimated that if the dose is 0.03mg then the probability of an adverse reaction is approximately 0.3, and if the dose is 0.05mg then the probability of an adverse reaction is approximately 0.6. Determine the values of m and b.

(b) [5 pts] Use your results to determine the function that represents the probability of an adverse reaction given the dosage. If you are unsure of your results in the previous part assume $m = 50$ and $b = -2$. (Explicitly state you are using these values.)

[1] Michael E. Ginevan, Deborah K. Watkins, Logarithmic dose transformation in epidemiologic dose–response analysis: Use with caution, Regulatory Toxicology and Pharmacology, Volume 58, Issue 2, 2010, Pages 336-340, https://doi.org/10.1016/j.yrtph.2010.07.007

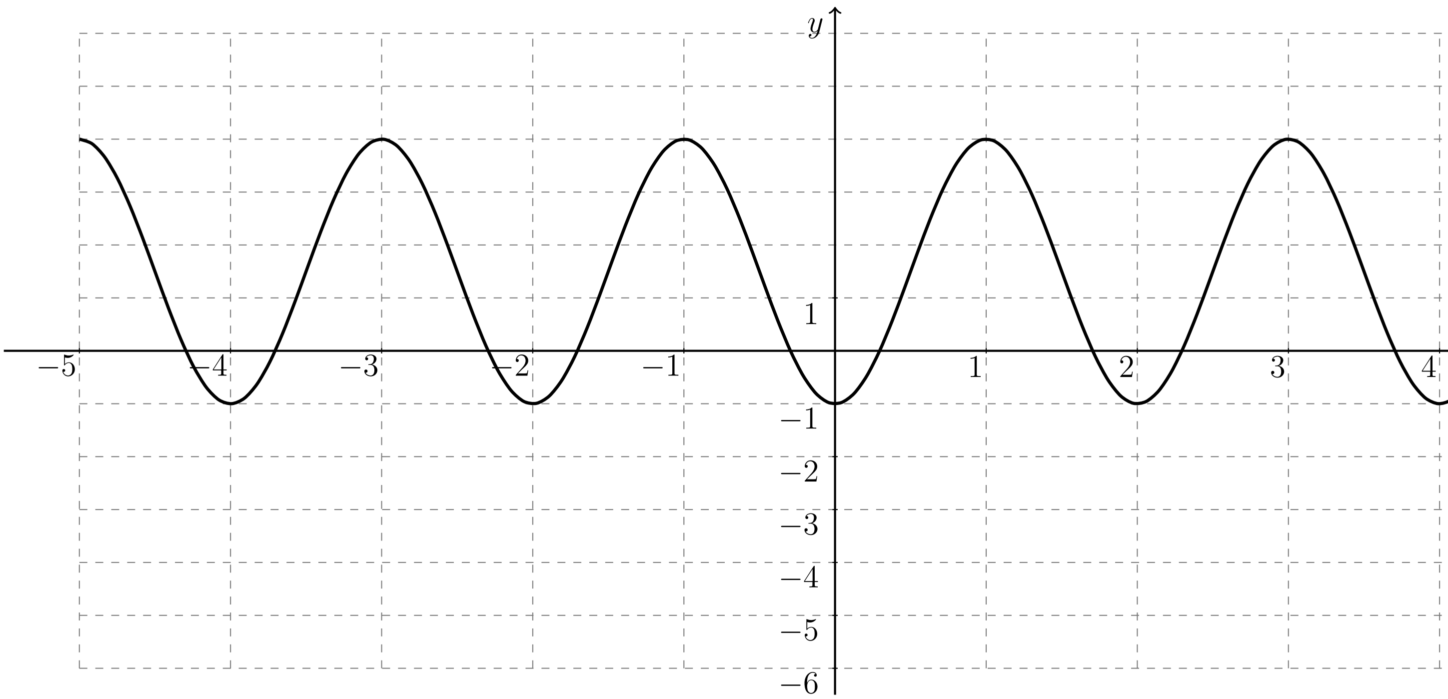
12. [10 pts] Verify the following identity,

$$(\cos(\alpha) - \cos(\beta))^2 + (\sin(\alpha) - \sin(\beta))^2 \quad = \quad 2 - 2\cos(\alpha - \beta)$$

13. [10 pts] Determine a formula for the function whose graph is shown below expressed as a cosine function,

$$k(x) \quad = \quad A\cos(bx+c)+d.$$

The values of A and b should be positive numbers.



$A =$

$b =$

$c =$

$d =$

14. [10 pts] There are 12 hours of daylight available to a bird. Suppose, the bird only takes part in two activities, foraging and defending its territory. The bird will fill the whole 12 hours with only those two activities, and each activity has an energy cost associated with it:

Foraging The energy cost of foraging is 7 times the amount of time (in hours) spent foraging (i.e. $7 *$time foraging). For example, if the bird spends 3 hours foraging the energy cost is 21 energy units.

Defending Territory To defend its territory the bird must fly around a large area, and the energy cost of defending its territory is the square of the time spent defending its territory (i.e. $(\text{time defending})^2$). For example, if the bird spends three hours defending its territory, the cost is 9 units.

Determine the amount of time the bird should spend foraging and the time defending its territory that will **minimize** the bird's total energy cost.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): ____________ Instructor (print): ____________ Time: ____________