Section 2.4C: More Building Functions.

Functions from 2D Geometry, continued.

1. Two cars leave an intersection at the same time. Car A travels north at 25 miles per hour, and Car B travels east at 30 miles per hour. Find the distance (in miles) between the two cars at time \( t \), where \( t \) represents the number of minutes elapsed since the cars left the intersection.

2. A square is inscribed with another square of a side length \( x \) as shown below. If the length of \( BC \) is 10 times the length of \( AB \), express the area of the inscribed square as a function of \( x \).

3. The relative positions of an aircraft runway and an \( a = 10 \)-foot-tall control tower are shown in the figure. The beginning of the runway is at a perpendicular distance of \( b = 300 \) feet from the base of the tower. If \( x \) denotes the distance an airplane has moved down the runway, express the distance \( d \) between the airplane and the top of the control tower as a function of \( x \).

Functions from 3D Geometry.

3D Objects: Here are more geometric formulas that you'll need.

- **Sphere**: For a sphere of radius \( r \):
  
  \[ SA = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3 \]

- **Box**: For a box with length \( L \), width \( W \), and height \( H \):
  
  \[ SA = \text{sum of six faces} \quad V = L \cdot W \cdot H \]

- **Cylinder**: For a cylinder with radius \( r \) and height \( h \):
  
  \[ SA = 2\pi r^2 + 2\pi r \cdot h \quad V = \pi r^2 \cdot h \]

- **Cone**: For a cone with radius \( r \) and height \( h \):
  
  \[ SA = \pi r^2 + \pi r \cdot \sqrt{r^2 + h^2} \quad V = \frac{1}{3}r^2 \cdot h \]

1. The figure shows a box with a square base. The volume of the box is 380 cubic feet. Express the surface area of the box, including top and bottom, as a function of \( x \).

2. A right circular cylinder has radius \( r \) and height \( h \). The surface area of the cylinder, including top and bottom, is 460 sq. ft. Express the volume of the cylinder as a function of \( r \).

3. A right circular cylinder has radius \( r \) and height \( h \). The volume of the cylinder is 3700 cubic feet. The bottom of the cylinder is reinforced steel, costing $35 per square foot, whereas the sides and top cost just $5 per square foot. Express the total cost of the cylinder as a function of \( r \).

4. A silo is to be built as in the diagram, a cylinder surmounted by a hemisphere. The radius of the silo is \( r = 70 \) ft and the TOTAL silo height \( L \) is not yet determined. The cost to paint the outside of the entire silo is $12 per square foot. Express the cost to paint the silo as a function of \( L \).

5. A wind tunnel is formed of a half cylinder with a rectangular base and semicircular ends. The volume must be 10000 \( \text{ft}^3 \) and the curved roof costs $11 per \( \text{ft}^2 \) to build. Find an expression that represents the (total) cost to build the curved roof as a function of the radius \( r \) of the semicircular end.

6. A bowl in the shape of a hemisphere with radius \( R \) inches, is filled with water to a depth \( h \) inches as shown below. The area of the surface of the water is \( A = 27\pi \) square inches. Express the radius \( R \) of the bowl as a function of the depth of water \( h \).