NCTM’s Curriculum Focal Points for Grades PreK – 8

Sybilla Beckmann

Department of Mathematics
University of Georgia

Georgia Mathematics Conference, October 2007
Outline of today’s presentation

- What the Focal Points are
  - Background, orienting ideas to place the Focal Points in a context
  - Some activities and problems that fit with the Focal Points
  - Some ways the Focal Points are a foundation for later math (intertwined with activities/problems)
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Curriculum Focal Points
for Prekindergarten through Grade 8 Mathematics

A Quest for Coherence

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Sybilla Beckmann  (University of Georgia)
Focal Points:

- 3 Focal Points at each grade level, PreK – 8
- clusters of the most significant mathematical concepts and skills at each grade level, PreK – 8
- the vast majority of instructional time should be spent on the Focal Points

Connections to the Focal Points:

- related content, including contexts for the focal points
- continuing development of topics in focal points of previous grades
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### Grade 2 Curriculum Focal Points

**Number and Operations**: Developing an understanding of the base-ten numeration system and place-value concepts

Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).

**Number and Operations and Algebra**: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction

Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

**Measurement**: Developing an understanding of linear measurement and facility in measuring lengths

Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).

### Connections to the Focal Points

**Number and Operations**: Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition.

**Geometry and Measurement**: Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.

**Algebra**: Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.
Why do we need Focal Points?
US curricula are unfocused, US instruction is unfocused

US state math curriculum documents:

“The planned coverage included so many topics that we cannot find a single, or even a few, major topics at any grade that are the focus of these curricular intentions. These official documents, individually or as a composite, are unfocused. They express policies, goals, and intended content coverage in mathematics and the sciences with little emphasis on particular, strategic topics.”
Did reform lead to widening of curricula?

From *A Splintered Vision* (1997):

“Mathematics reform recommendations thus far seem to have affected curricula primarily by inclusion of additional topics, with a concomitant decrease in how focused these curricula are. This seems to result from our unwillingness to drop other topics when newer topics are added.”
Breaking the “mile-wide-inch-deep” habit

*Every* mathematical skill and concept

- has some useful application
- has some connection to other concepts and skills

So what mathematics should we focus on?
Statistics and probability are increasingly important in science and in the modern workplace.

Should we therefore focus on statistics and probability in school mathematics?

Calculators are always at hand (cell phones), so paper and pencil calculations seem nearly obsolete.

Should school mathematics no longer cover paper and pencil calculations?
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More students should be prepared to continue in math and science

From **Rising Above The Gathering Storm**: *Energizing and Employing America for a Brighter Economic Future* (2007):

“Having reviewed trends in the United States and abroad, the committee is deeply concerned that the scientific and technological building blocks critical to our economic leadership are eroding at a time when many other nations are gathering strength.”
More students should be prepared to continue in math and science
From *Rising Above the Gathering Storm*, 2007

“Recommendation A: Increase America’s talent pool by vastly improving K–12 science and mathematics education.”

“Action A–3: Enlarge the pipeline of students who are prepared to enter college and graduate with a degree in science, engineering, or mathematics . . .

“Action C–1: Increase the number and proportion of US citizens who earn bachelor’s degrees in the physical sciences, the life sciences, engineering, and mathematics . . .”
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Calculus provides a theoretical framework for many parts of physics and engineering, it is a foundation for additional mathematics and statistics, and it is used in chemistry and biology.
College math and beyond: calculus is foundational

Not all students will take calculus, not all students will go to college

BUT

at the end of 8th grade all options should be open to all students!
To be prepared for success in calculus, students must:

- be proficient with arithmetic
- be proficient with algebra:
  - algebraic manipulations
  - set up and solve equations
  - work with functions
- be proficient with geometry/measurement:
  - area and volume
  - Pythagorean theorem
  - similarity
  - trigonometry
What is needed to succeed in calculus?

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- solve multi-step problems
- be able to “take apart, analyze, put back together”
- visualize and draw simple pictures to capture a situation for mathematical analysis
- use logical reasoning and understand why methods work
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Some aspects of the Focal Points

- Solid preparation for future mathematics, in particular, for algebra
- Numbers & operations and geometric measurement are emphasized
- Algebra gradually receives more emphasis in the later grades
Some aspects of the Focal Points

Arithmetic:

- The need for quick recall of basic facts is stated explicitly
- Quick recall of basic facts is built on understanding relationships among facts
- Algorithms of arithmetic:
  - develop them
  - understand why they work in terms of place value and properties of operations
  - develop fluency
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Geometry/measurement:

- composing and decomposing is emphasized
- understanding area and volume formulas by composing and decomposing (links to arithmetic and algebra)
- use area and volume formulas to solve problems
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Overall: a blend of skills and understanding
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Overall: a blend of skills and understanding
Adding It Up

Mathematical Proficiency:
- conceptual understanding
- procedural fluency
- strategic competence
- adaptive reasoning
- productive disposition
Prekindergarten
Why PreK?

Mathematics is predictive of later mathematics and literacy/reading, whereas literacy is predictive only of later reading.

- Equity: gaps between income groups and between nations may be as wide at 3–4 years of age as at the elementary level.
- Curricula/programs using research-based developmental progressions of mathematical concepts and skills close gaps; lower-income children can even outperform their middle-class counterparts.
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Some PreK, K, Grade 1 Ideas
From recognizing small amounts to “counting on”

Show briefly:
Some PreK, K, Grade 1 Ideas
From recognizing small amounts to “counting on”

Then hide:

![Diagram of a square]

Ask: How many are there? (Show with your fingers.)
Some PreK, K, Grade 1 Ideas
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Show briefly:
Some PreK, K, Grade 1 Ideas
From recognizing small amounts to “counting on”

Then hide:

Ask: How many are there?
A $5 + \square = 7$ problem:

Maya has 5 beads. She needs 7. How many more beads does Maya need?

Children can solve this by counting on from 5:

"Already 5"  "so 2 more"
Some PreK, K, Grade 1 Ideas
From recognizing small amounts to “counting on”

A \(7 - 5 = \square\) problem:

There were 7 nuts. Then a mouse ate 5. How many nuts are left?

Children can also solve this by counting on from 5:

“I took away 5”  
\[\begin{array}{cc}
6 & 7 \\
\hline
5 & 0 \\
\end{array}\]

“so 2 are left”
Children break numbers apart:

\[ 7 = 2 + 5 \]

\[ 7 = 3 + 4 \]
The **make-a-ten strategy** relies on breaking numbers apart and implicitly uses the associative property of addition:

$$8 + 6$$
Grades 1 and 2
Basic facts, make-a-ten strategy, and regrouping

The **make-a-ten strategy** relies on breaking numbers apart and implicitly uses the associative property of addition:

\[
\begin{array}{cc}
8 + 6 & \\
\downarrow & \\
2 & 4
\end{array}
\]

\[
\begin{array}{cc}
\text{ Eight dots } & \text{ Six dots } \\
\text{ Two dots } & \text{ Four dots }
\end{array}
\]

The make-a-ten strategy relies on breaking numbers apart and implicitly uses the associative property of addition:

$$8 + 6 = 8 + (2 + 4)$$
The **make-a-ten strategy** relies on breaking numbers apart and implicitly uses the associative property of addition:

\[ 8 + 6 = 8 + (2 + 4) = (8 + 2) + 4 = 14 \]
Grades 1 and 2
Basic facts, make-a-ten strategy, and regrouping

Subtracting by regrouping: taking away from 10

13 - 9

[Diagram of 13 dots in a vertical line, with 4 dots removed]
Grades 1 and 2
Basic facts, make-a-ten strategy, and regrouping

Subtracting by regrouping: taking away from 10

13 - 9

10 3
Subtracting by regrouping: taking away from 10

13 - 9

10 3

take 9 from 10
Grades 1 and 2
Basic facts, make-a-ten strategy, and regrouping

Subtracting by regrouping: taking away from 10

\[ 13 - 9 \]
\[
\begin{array}{c}
10 \quad 3 \\
\end{array}
\]

- take 9 from 10
- 1 and 3 make 4
Subtraction with regrouping

\[
\begin{array}{c}
62 \\
- 45 \\
\hline
\end{array}
\]
Subtraction with regrouping

\[
\begin{array}{r}
62 & \underline{\ | \ | | \ |} \\
-45 & 4 \ 5 \\
\end{array}
\]
Subtraction with regrouping

\[
\begin{array}{c}
510 \\
62 \\
\text{+}
\end{array}
\]

\[
- 45 \\
4 \\
5
\]

\[
\begin{array}{c}
\text{=}
\end{array}
\]

\[
17
\]
Relating basic facts to other basic facts builds a foundation for quick recall and uses properties of arithmetic

\[4 \times 7 = 4 \times 5 + 4 \times 2\]
Relating basic facts to other basic facts builds a foundation for quick recall and uses properties of arithmetic

\[ 4 \times 7 = 4 \times 5 + 4 \times 2 \]

\[ 4 \times (5 + 2) = 4 \times 5 + 4 \times 2 \]
Grades 3 and 4
Multiplication basic facts and the multiplication algorithm

\[
\begin{array}{c}
10 \\
+ \\
3
\end{array}
\quad \quad
\begin{array}{c}
10 \times 10 \\
+ \\
3 \times 10
\end{array}
\quad \quad
\begin{array}{c}
10 \times 4 \\
+ \\
3 \times 4
\end{array}
\quad \quad
\begin{array}{c}
14 \\
\times 13
\end{array}
\quad \quad
\begin{array}{c}
12 \\
30 \\
40 \\
100
\end{array}
\quad \quad
\begin{array}{c}
182
\end{array}
\]
4th graders develop understanding of why multiplication procedures work based on

- what multiplication means
- place value
- the distributive property (decomposing and composing)
Assertions should have reasons

From NCTM *Principles and Standards for School Mathematics* (PSSM), 2000:

“From children’s earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons. Questions such as “Why do you think it is true?” and “Does anyone think the answer is different, and why do you think so?” help students see that statements need to be supported or refuted by evidence.” (chapter 3)
Reasoning is essential to understanding math

From PSSM:

“Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense.”
(chapter 3)
Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study

<table>
<thead>
<tr>
<th>Country</th>
<th>Percentage</th>
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</thead>
<tbody>
<tr>
<td>Australia</td>
<td>25%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>25%</td>
</tr>
<tr>
<td>Hong Kong SAR</td>
<td>20%</td>
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<tr>
<td>Czech Republic</td>
<td>10%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>10%</td>
</tr>
<tr>
<td>United States</td>
<td>0%</td>
</tr>
</tbody>
</table>
Understanding *why* the multiplication algorithm works connects arithmetic to algebra.

\[(x + 3)(x+4) = x^2 + 3x + 4x + 3\cdot4\]
The Focal Points ask students not just to know and be able to use area and volume formulas but also to develop an understanding of where these formulas come from by decomposing and composing.

Note the parallel to understanding arithmetic by decomposing and composing!
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Note the parallel to understanding arithmetic by decomposing and composing!
One method:
Explaining why the area formula for triangles is valid

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Explaining why the area formula for triangles is valid

One method:

\[ \text{Area of triangle} = \frac{1}{2} \times b \times h \]

Where:
- \( h \) is the height of the triangle
- \( b \) is the base of the triangle

The area formula for a triangle can be derived by dividing the triangle into two congruent triangles and rearranging them to form a parallelogram.
Another method:
Another method:
Another method:

\[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]
What is the area of the pink triangle inside the square?
volume = (height) × (area of base)
Decomposing into slices in integral calculus

Derive the formula for the volume of a cone of radius \( r \) and height \( h \)

By similar triangles,

\[
\frac{y}{x} = \frac{r}{h}
\]

so

\[
\text{Volume} = \int_{0}^{h} \pi y^2 \, dx = \int_{0}^{h} \pi \left( \frac{r}{h} \right)^2 x^2 \, dx = \pi \frac{r^2}{h^2} \cdot \frac{1}{3} x^3 \bigg|_{0}^{h} = \frac{1}{3} \pi r^2 h
\]
Reasoning about ratio, rate, proportion, and similarity in the 6th, 7th, and 8th grade Focal Points

- 6th: Connecting ratio and rate to multiplication and division
- 7th: Developing an understanding of and applying proportionality, including similarity
- 8th: Proportional relationships are special cases of linear relationships; the slope of a line is a constant rate of change
A ratio problem: Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?

6th Grade, 2nd Focal Point:

- Ratios and rates derive from, and extend, pairs of rows in the multiplication table
- Students analyze simple drawings that indicate the relative sizes of quantities (applying multiplication and division)
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6th Grade, 2nd Focal Point:

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How many pails of each color to make 30 pails of green paint?

<table>
<thead>
<tr>
<th># of batches</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
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<tbody>
<tr>
<td># pails blue paint</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
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<tr>
<td># pails yellow paint</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
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<td># pails green paint produced</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
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</table>
Graphing equivalent ratios in a ratio table

<table>
<thead>
<tr>
<th># pails blue paint</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
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Focal Points
Connecting ratio tables to equivalent fractions

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<th># of batches</th>
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\[
\frac{2}{5} = \frac{?}{100}
\]
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<td># pails yellow paint</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>?</td>
</tr>
<tr>
<td># pails green paint produced</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
\frac{2}{5} = \frac{?}{100}
\]

\(\times 20\)
Connecting ratio tables to equivalent fractions

<table>
<thead>
<tr>
<th># of batches</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td># pails blue paint</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td># pails yellow paint</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td># pails green paint produced</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
\frac{2}{5} = \frac{40}{100} \times 20 \quad \text{or} \quad \frac{20}{100} = \frac{1}{5} \times 20
\]
Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?
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5 equal parts make 30 pails
Reasoning about ratio and proportion

- Rows in the multiplication table $\rightarrow$ equivalent ratios
- Tables of equivalent ratios $\rightarrow$ tables for functions
- Reasoning about unknown quantities in simple pictures $\rightarrow$ reasoning about unknown letters
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Reasoning about ratio and proportion

- Rows in the multiplication table → equivalent ratios
- Tables of equivalent ratios → tables for functions
- Reasoning about unknown quantities in simple pictures → reasoning about unknown letters
Grade 7
Reasoning about similar shapes

2 ft

3 ft

5 ft

?
One method:

2 ft $\times 2\frac{1}{2} = 5$ ft

3 ft $\times 2\frac{1}{2} = ?$
One method:

\[ \times 2\frac{1}{2} \]

2 ft

3 ft

\[ \times 2\frac{1}{2} \]

5 ft

7\frac{1}{2} ft
Another method:

\[ \times 1\frac{1}{2} \]

2 ft

3 ft

5 ft

?
Another method:

\[ \times 1\frac{1}{2} \]

2 ft \[ \times 1\frac{1}{2} \]

3 ft \[ \times 1\frac{1}{2} \]

5 ft

7\frac{1}{2} ft

related to trigonometric functions!
A line that crosses parallel lines creates congruent angles:
What do you notice about the angles?

$m \angle ABC = 134^\circ$

$m \angle BCD = 80^\circ$

$m \angle CDE = 146^\circ$
What do you notice about the angles?

- $\angle ABC = 133^\circ$
- $\angle BCD = 100^\circ$
- $\angle CDE = 127^\circ$

Parallel lines
What do you notice about the angles?

Parallel lines

$m\angle ABC = 166^\circ$

$m\angle BCD = 70^\circ$

$m\angle CDE = 124^\circ$
Without measuring the angles, explain why the sum of the angles at $B$, $C$, and $D$ must be $360^\circ$. (See www.math.uga.edu/~sybilla for hints.)
Concluding thoughts

- Arithmetic may *seem* obsolete but the reasoning within it is a foundation for all of mathematics.
- Geometry provides many opportunities for reasoning about decomposing and composing – a skill needed throughout mathematics.
- *Reasoning* about quantities (rather than going straight to “black box” procedures) provides mathematical connections.
- The Focal Points focus on reasoning and skill in arithmetic, geometry, and algebra and are a foundation for the further study of math and science.
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For hints on the angle problem and the triangle area problem, go to:

www.math.uga.edu/~sybilla