ANOTHER PROOF OF CAUCHY'S GROUP THEOREM

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Since $ab = 1$ implies $ba = (ab)b^{-1} = 1$, the identities are symmetrically placed in the group table of a finite group. Each row of a group table contains exactly one identity and thus if the group has even order, there are an even number of identities on the main diagonal. Therefore, $x^2 = 1$ has an even number of solutions.

Generalizing this observation, we obtain a simple proof of Cauchy’s theorem. For another proof see [1].

CAUCHY’S THEOREM. If the prime $p$ divides the order of a finite group $G$, then $G$ has $kp$ solutions to the equation $x^p = 1$.

Let $G$ have order $n$ and denote the identity of $G$ by $1$. The set

$$S = \{(a_1, \ldots, a_p) \mid a_i \in G, a_1a_2 \cdots a_p = 1\}$$

has $n^{p-1}$ members. Define an equivalence relation on $S$ by saying two $p$-tuples are equivalent if one is a cyclic permutation of the other.

If all components of a $p$-tuple are equal then its equivalence class contains only one member. Otherwise, if two components of a $p$-tuple are distinct, there are $p$ members in the equivalence class.

Let $r$ denote the number of solutions to the equation $x^p = 1$. Then $r$ equals the number of equivalence classes with only one member. Let $s$ denote the number of equivalence classes with $p$ members. Then $r + sp = n^{p-1}$ and thus $p \mid r$.

Reference


A REMARK ON BOUNDED FUNCTIONS

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Denote by $E$ the class of functions regular and bounded by unity in $|z| < 1$. Denote by $E^*$ the subclass of functions of $E$ which are in addition univalent in $|z| < 1$. Analogies of various inequalities which are known to hold for functions in the class $E$ have been obtained for functions of the class $E^*$. For example, it is known [3] that there exist functions in $E$ for which the sequence $\{a_0 + \cdots + a_n\}$ $f(z) = \sum a_nz^n$ is unbounded. On the other hand, it is shown by Fejér in [1] that if $f \in E^*$ then $|a_0 + \cdots + a_n| < 1 + (1/\sqrt{2})$ for all $n$. 119