CORRIGENDUM:
THE MODULAR CURVE $X_0(169)$ AND RATIONAL ISOGENY

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The error arose from equation (2) (in [2]) which should have read

$$XY\{X^{12} + X^{11}Y + \ldots + Y^{12} + \ldots + 124852(X + Y) + 15145\} - 13 = 0.$$ (2)

Consequently, the only possible values of $X$ and $Y$ modulo 3, rational over $\mathbb{F}_3$, are $X \equiv \pm 1(3)$ and $X \equiv Y(3)$. Since $W = W_{169}$ permutes $X$ and $Y$, these two points are fixed by $W$. The modular invariant corresponding to them is the supersingular invariant $j = 0$ in characteristic 3.

As in [1], we calculated the characteristic polynomial of the Hecke operator $T_p$ for $p = 2$ and 3. The polynomial for $T_2$ is $(X^3 - 2X^2 - X + 1)(X^3 + 2X^2 - X - 1)(X^2 - 3)$, and that for $T_3$ is $(X^3 + 2X^2 - X - 1)^2(X - 2)^2$. From these it is clear that the Eisenstein quotient $J^{(7)} = J_0(169)$ has Mordell–Weil group of order 7 over $\mathbb{Q}$ which is generated by the image of the class of the divisor $P_0 - P$. Also the reduction homomorphism on $J^{(7)}(\mathbb{Q})$ is injective modulo 2, 3 and 13.

Let $P \in J_0(169)(\mathbb{Q})$. Since the elliptic curve in a rational pair $(E, A)$ corresponding to $P$ has potentially good reduction modulo 3, the image of the divisor class $P - WP$ on $J^{(7)}$ reduces to 0 modulo 3. Hence $P$ cannot reduce to $P_0/\mathbb{F}_2$ or $P_0/\mathbb{F}_3$ modulo 2, or to $P_0/\mathbb{F}_2$, $P_0/\mathbb{F}_3$, or $P_0/\mathbb{F}_3$ modulo 13.

Consequently, the only possibilities for $X$ and $Y$ are

(i) $X = Y = \pm 13^r/m$,

(ii) $X = 13Y = \pm 13^r/m$,

where $\epsilon = \pm 1$, $r$ is an integer and $m$ is a positive integer divisible only by primes $p$ congruent to 1 modulo 13. Both cases are easily dismissed.

References


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