

MATH 4400/6400 2009 MIDTERM QUIZ

Directions: Do all four problems. **No calculators are permitted.**

1) Find all integer solutions to the following Diophantine equations:

a) $6x + 15y = 33$.

b) $21580989218526x + 9821375928754y = 12935870219857209385709187$.

2) Compute these Legendre symbols (the denominators are all prime numbers):

$$\left(\frac{-1}{3571}\right), \left(\frac{200}{2819}\right), \left(\frac{571}{229}\right).$$

3) Show that $\log_3 5$ is irrational.

(Suggestion: A good beginning is “Suppose for a contradiction that there exist positive integers a and b such that $\log_3 5 = \frac{a}{b}$. Then $3^{\frac{a}{b}} = 5$.”)

4) Determine exactly which integers N with $1,000,001 \leq N \leq 1,000,020$ are sums of two squares. For your convenience, the prime factorizations of many of the integers in this range are given below. (You should figure out why you don't need to see the prime factorizations of the others!)

$$1,000,001 = 101 \cdot 9901, \quad 1,000,002 = 2 \cdot 3 \cdot 166667, \quad 1,000,004 = 2^2 \cdot 53^2 \cdot 89.$$

$$1,000,005 = 3 \cdot 5 \cdot 163 \cdot 409, \quad 1,000,006 = 2 \cdot 7 \cdot 71429, \quad 1,000,008 = 2^3 \cdot 3^2 \cdot 17 \cdot 19 \cdot 43.$$

$$1,000,009 = 293 \cdot 3413, \quad 1,000,010 = 2 \cdot 5 \cdot 11 \cdot 9091, \quad 1,000,012 = 2^2 \cdot 13 \cdot 19231.$$

$$1,000,013 = 7 \cdot 373 \cdot 383, \quad 1,000,014 = 3 \cdot 166669, \quad 1,000,016 = 2^4 \cdot 62501.$$

$$1,000,017 = 3^2 \cdot 23 \cdot 4831, \quad 1,000,018 = 2 \cdot 500009, \quad 1,000,020 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 2381.$$