

MATH 4400/6400 2009 MAKEUP MIDTERM QUIZ

Directions: Do all four problems. **No calculators are permitted.**

1) Find all integer solutions to the following Diophantine equations:

a) $51x + 85y = 34$.

b) $21580989218945x + 9821375928990y = 12935870219857209385709187$.

2) Compute these Jacobi symbols:

$$\left(\frac{-1}{4409}\right), \left(\frac{338}{2819}\right), \left(\frac{571}{313}\right).$$

3) a) State the full Two Squares Theorem, which gives necessary and sufficient conditions for a positive integer to be the sum of two integer squares.

b) Deduce from the Two Squares Theorem that any integer $n \equiv 3 \pmod{4}$ is not a sum of two squares. (Remark: Usually I do not care if you solve a problem using a different technique than is suggested. But this time I really do want you to do it this way, which is probably harder than, say, looking modulo 4.)

4) For each integer N with $100 \leq N \leq 120$, determine whether N is a sum of two squares.