# Graduate Student Mock AMS Conference 2015

*July 29 – July 31*

# Abstracts

## Brian Bonsignore

## Introduction to Goodwillie's Calculus of Functors

### Abstract

I'll give a brief introduction to Goodwillie's approach for successively approximating certain functors $F$ in homotopy theory. This method produces a tower of functors $F \to \cdots T_k F \to T_{k-1} \to \cdots \to T_0 F$ and is considered analogous to the approximation of smooth functions by Taylor polynomials in calculus. I'll define the relevant terminology, e.g. what a "polynomial functor" is, and indicate how the framework is used to study the space of embeddings of one smooth manifold to another.

## Harrison Chapman

## Asymptotics of random link diagrams

### Abstract

As an alternative to random polygon models of knotting, we consider random knots and links by sampling random link diagrams. Link diagrams are decorated embedded graphs (called *maps*) whose study is simplified by choosing a *root*; a single marked and directed edge of the diagram. We prove that for different classes of rooted diagrams, any sufficiently nice substructure (called a *tangle*) occurs "often" almost certainly in a randomly selected diagram. As a key consequence, we show that these results for rooted diagrams apply to usual link diagrams.

## Riley Ellis

## Six Points in $\mathbb{C}P^1$

### Abstract

In Algebraic Geometry, a moduli space is an algebraic variety that parameterizes some class of geometric objects, up to some equivalence relation. One method of constructing these is to take a parameter space and quotient by a group of automorphisms. However, in the same way that a topological quotient of a manifold may not be a manifold, the "naive" way of taking quotients in Algebraic Geometry often produces an object that is not a variety. Geometric Invariant Theory is one method of forming these "quotients" with reasonable properties (like being a variety). I will discuss these general ideas and how they relate to constructing the moduli space of configurations of six points in $\mathbb{C}P^1$.

## Saurabh Gosavi

## Severi-Brauer varieties and Central Simple Algebras

**Abstract**

A Severi-Brauer variety is a variety which becomes isomorphic to a projective space after a suitable base extension. In this talk we will quickly prove that the set of isomorphism classes of central simple algebras are in one to one correspondence with the set of isomorphism classes of Severi-Brauer varieties. In addition, we will show that a necessary and sufficient condition for a central simple algebra to be split is that the corresponding Severi-Brauer variety has a rational point.

## Ernest Guico

## A Talk Riddled With Codes

**Abstract**

Fifteen people are seated in a room and a hat is placed on each person's head. The color of each person's hat, which may be either black or white, is decided by tossing a fair coin, and each person can see all hats except their own. Without any form of communication, at least one person must guess the color of their own hat. Everyone who wishes to guess must guess at the same time. The group wins if at least one person guesses correctly and no one guesses incorrectly. This talk will discuss strategies that such a group might utilize. In particular, we will focus on a surprisingly effective strategy with ties to the field of algebraic coding theory.

## Jacob Hicks

## Quadratic Forms and the Geometry of Numbers

**Abstract**

Representation theorem for quadratic forms have been proven in many different ways. This talk is about using various Geometry of Numbers to create small multiple theorems that can combined with computational techniques to prove representation theorem for quadratic forms.

## Natalie Hobson

## An Brief Introduction to Schubert Calculus

**Abstract**

How many lines in $\mathbb{P}^3$ contain two given points? Or maybe more challenging, how many lines intersect four given lines in $\mathbb{R}^3$? These types of questions can be answered using Schubert calculus. To answer such questions we will define and discuss the Grassmannian variety. We will give a structure of this variety in terms of Schubert cells. Using this structure we will answer the previously stated questions in a beautiful and explicit computational way.

## Lauren Huckaba

## Some Variants of the Erdős Distinct Distances Problem


### Abstract

In 1946, Erdős asked: What is the least number of distinct distances determined by $n$ points in the plane? This is a long-standing problem that was essentially solved in 2010 by Larry Guth and Nets Katz. In this talk, we discuss some variants of the distinct distances problem.


## Kenneth Jacobs

## Reduction of Rational Maps


### Abstract

This talk will present the concept of the reduction of a rational map defined over a global field. We will introduce an analytic tool – the crucial measures – that can be used to characterize the reduction types of the map under change of coordinates.


## Jason Joseph

## Zero Free Intervals of Chromatic Roots


### Abstract

Chromatic polynomials were introduced by Birkhoff in 1912 in an attempt to solve the 4-color problem. For a finite graph $G$, there exists a polynomial $P(G, \lambda)$ which, when evaluated at a positive integer $k$, gives the number of proper colorings of $G$ using $k$ colors. Birkhoff and Lewis conjectured that for planar graphs, $P(G, \lambda)$ has no real roots in $[4, \infty)$, which would have implied the 4-color theorem. However, they were only able to show the absence of chromatic roots in the interval $[5, \infty)$. Although the existence of chromatic roots for planar graphs in $(4, 5)$ is still unknown, it is known that chromatic roots in general are absent from the intervals $(-\infty, 0)$, $(0, 1)$, $(1, \frac{32}{27}]$, and that they are dense in the real interval $[\frac{32}{27}, \infty)$ and the complex plane. These results and some general properties of chromatic polynomials will be explored.


## Allan Lacy Mora

## Splitting fields of genus one curves


### Abstract

I will report on current work based on the following question: given a (genus one) curve C defined over a ground field K, what are the smallest field extensions of K over which C has points? I will focus on the case where C is given as a binary quartic and K is a Henselian discrete valuation field.

## Jay Lanterman

## Generalized Barycentric Coordinates


### Abstract

Barycentric coordinates are well-known as a convenient way to express points in triangles as a weighted average of the triangle's vertices, but the concept can be extended (albeit non-uniquely) in order to similarly express points in an arbitrary polygon in terms of the polygon's vertices. We'll discuss a couple of possible coordinate choices, which (like standard barycentric coordinates) are derived from some geometric properties of the polygon, along with a brief introduction to barycentric function interpolation.


## Phong Luu

## Stochastic Approximation Algorithms


### Abstract

If $f(.)$ and its gradient can be observed without error at any desired values, then classical numerical methods such as Newton-Raphson method among others can be applied to to find the root of $f(.)$. However, these methods cannot perform well when the function is not known but the "noisy" measurements at any desired values. To solve this problem, we use stochastic recursive algorithms, also known as stochastic approximations. We will discuss the stochastic approximation algorithms introduced by Robbins and Monro and by Kiefer and Wolfowitz.


## Patrick K. McFaddin

## Milnor-Witt Groups and Homotopy Theory of Schemes


### Abstract

MIlnor-Witt $K$-groups of a field $F$, defined by F. Morel, are given by a sequence of groups which encode information about the arithmetic and admissible quadratic forms over $F$. These groups are closely related to Milnor $K$-theory, and admit an explicit description in terms of generators and relations. Milnor-Witt groups are a fundamental ingredient in the area of motivic homotopy theory, arising as the homotopy (sheaves of) groups of punctured affine space. In this talk, I will introduce Milnor-Witt groups of fields, extend this definition to Milnor-Witt sheaves of groups, and give a brief description as to how these groups fit into the framework of motivic homotopy theory.


## Marko Milosevic

## Field of Definition of Torsion Points on Elliptic Curves


### Abstract

The torsion of elliptic curves has been studied with the torsion points lying in the same field as the field of definition of the elliptic curve. New results by Alvaro Lozano-Robledo look at torsion points of elliptic curves defined solely over the rationals. Consider the set $S_{\mathbb{Q}}(d)$ of primes $p$ for which there exists a number field $K$ of degree less than $d$ and an elliptic curve $E/\mathbb{Q}$, such that the order of the torsion subgroup $E(K)$ is divisible by $p$. We will give some of the results on the bounds of $S_{\mathbb{Q}}(d)$.

## Tom Needham

## Framed Loops and Infinite-Dimensional Grassmannians


### Abstract

Curvature and torsion are real-valued functions that give convenient coordinates on the infinite-dimensional manifold of isometry classes of curves in Euclidean space. This coordinate system comes with the cost that we can no longer easily tell whether a curve is closed—finding effective conditions on curvature and torsion that ensure that a curve closes is one of the oldest problems in elementary differential geometry.

In this talk we will describe an alternative isometry-invariant coordinate system that uses a pair of complex-valued functions to describe a space curve with an arbitrary framing. In these coordinates, the closure condition is simply $L^2$ orthogonality. This allows us to construct a Riemannian metric on the moduli space of closed framed curves, up to Euclidean similarity. In fact, the moduli space of framed curves is a Kähler manifold, which is isometric to an infinite-dimensional complex Grassmannian. The complex structure is related to the symplectic structure on the space of unparameterized loops, which was introduced by Marsden and Weinstein as a tool to study vortex filaments. We will describe these geometric structures as well as some rather concrete applications to computer graphics and protein shape recognition.


## Hans Parshall

## Long arithmetic progressions of twin primes


### Abstract

We'll sketch how one can combine the Maynard-Tao method with ideas of Green-Tao and Pintz to (almost) find arbitrarily long arithmetic progressions of twin primes. We'll also describe how an analogous method can find arbitrarily large affine subspaces of prime polynomials over a finite field that are separated by low degree polynomials.


## Eric Perkerson

## A Mathematical Introduction to Bayesian Statistics


### Abstract

Bayesian statistics is an approach to statistics relying heavily on Bayes' Theorem, which relates a conditional probability $P(A|B)$ to the reverse conditional probability $P(B|A)$. Unique to this approach to statistics is the use of a prior distribution, which incorporates information known prior to the collection of a particular data set being analyzed. As an introduction, I will first talk about the foundations of probability and statistics before covering the basics of Bayesian statistics.

## Luca Schaffler

## The Secondary Polytope


### Abstract

After a brief introduction to convex geometry, we define the concept of secondary polytope associated to a marked polytope. We explore some of the main properties of the secondary polytope and we discuss a concrete example of this: the Stasheff polytope. Finally, we discuss an application of this theory to the study of the moduli space of stable toric pairs in algebraic geometry.


## George Slavov

## Recursive Refinement of Polygons


### Abstract

The finite element method is a popular technique for finding numerical solutions to partial differential equations. For this talk we consider boundary value problems on a polygonal domain $\Omega \subset \mathbb{R}^2$. The method requires that $\Omega$ be triangulated. The refinement of the triangulation is then a means of improving the numerical accuracy of the solution. Finite elements have been defined on quadrilateral partitions of polygons and more recently over arbitrary polygonal partitions using so-called virtual elements. A natural need arises for schemes with which to refine polygonal partitions. In order to keep code complexity to a minimum, a partition should ideally consist only of pentagons or only of hexagons or in general $n$-gons. In this talk, however, I will show that there is a fundamental barrier: if $n > 5$, then a partition of an $n$-gon into $n$-gons of smaller size does not exist.


## Lee Troupe

## Bounded gaps between primitive polynomials in $\mathbb{F}_q[t]$


### Abstract

A famous conjecture of Artin states that there are infinitely many prime numbers for which a fixed integer $g$ is a primitive root, provided $g \neq -1$ and $g$ is not a perfect square. Thanks to work of Hooley, we know that this conjecture is true, conditional on the Generalized Riemann Hypothesis. Using a combination of Hooley's analysis and the techniques of Maynard and Tao used to prove the existence of bounded gaps between primes, Pollack has shown that there are bounded gaps between primes with a prescribed primitive root, again conditional on GRH. In this talk, we discuss the analogue of Pollack's work in the function field case; namely, that given a monic polynomial $g(t)$ which is not an $\ell$th power for any $\ell$ dividing $q - 1$, there are bounded gaps between monic irreducible polynomials $P(t)$ in $\mathbb{F}_q[t]$ for which $g(t)$ is a primitive root (which is to say that $g(t)$ generates the group of units modulo $P(t)$). In particular, we obtain bounded gaps between primitive polynomials, corresponding to the choice $g(t) = t$.

## Maren Turbow

## Structure Theory of Graded Central Simple Algebras


### Abstract

In this talk we will begin by discussing basic definitions related to graded central simple algebras and provide examples. We will then look at some structure theorems for even and odd graded central simple algebras, graded by $\mathbb{Z}/n\mathbb{Z}$.


## Abraham Varghese

## If Karajan is to Music, What is Swan to Math?


### Abstract

The talk will be about some conductors of the math world!


## Lori Watson

## An Introduction to P-orderings


### Abstract

In this talk, we will give a brief introduction to Manjul Bhargava's notion of a P-ordering of an arbitrary subset of a Dedekind ring. We will discuss some of the basic properties of a P-ordering and highlight a few applications.


## Matt Zawodniak

## Lecturing Left on the Cutting Room Floor: A Video Project for Pre-service Teachers.


### Abstract

Students have been shifting their education from traditional classrooms to electronic ones through many online programs and universities. According to the U.S. Department of Education, over 2.5 million students were enrolled in exclusively distance education courses in 2012, and those numbers continue to climb. Even in lieu of strictly taking classes online, students are turning towards YouTube to gain knowledge, supplement classwork, or deepen their understanding of content. As of April 2015 educational YouTube channels such as SciShow and MinutePhysics have over 2.5 million subscribers apiece, indicating that they are effective means of STEM communication. Knowing that there is such a vast resource of educational material online, students seek out videos to enhance, complement, or replace their classroom learning. For these reasons, it is imperative that new teachers entering the workforce have some experience with creating and using online educational tools. To give pre-service teachers some experience with this medium, a video assignment was developed and implemented in two successive terms of an Arithmetic and Problem Solving for Elementary School Teachers course at the University of Georgia. This talk will focus on the assignment itself, how it developed across the two terms, and how students felt about the assignment in each iteration. Time permitting, some examples of the projects will be shown.

## The Wake/Davidson Experience in Number Theory Research, Group I:


## Alexi Block Gorman, Tyler Genao, Heesu Hwang, Noam Kantor, Sarah Parsons


## The density of primes dividing a certain non-linear recurrence sequence


### Abstract

Define the sequence $\{b_n\}$ by $b_0 = 1$, $b_1 = 2$, $b_2 = 1$, $b_3 = -3$ and

$$b_n = \begin{cases} \frac{b_{n-1}b_{n-3}-b_{n-2}^2}{b_{n-4}} & \text{if } n \not\equiv 2 \pmod 3 \\ \frac{b_{n-1}b_{n-3}-3b_{n-2}^2}{b_{n-4}} & \text{if } n \equiv 2 \pmod 3. \end{cases}$$

We relate the sequence $\{b_n\}$ to the coordinates of points on the elliptic curve $E : y^2 + y = x^3 - 3x + 4$. We use Galois representations attached to $E$ to prove that the density of primes dividing some term in the sequence is $\frac{179}{336}$.


## The Wake/Davidson Experience in Number Theory Research, Group II:


## Sarah Blackwell, Gabe Durham, Tiffany Treece


## A generalization of Mordell to ternary quadratic forms


### Abstract

A 1958 paper of Mordell gives a proof of which integers are representable as the sum of three squares using highly creative, almost magical congruence conditions. We have modified this argument to say what values are represented by six (and counting!) additional ternary quadratic forms. We will discuss the general ingredients of this method, when it appears to succeed as well as when it is guaranteed to fail, and current ideas for future work.